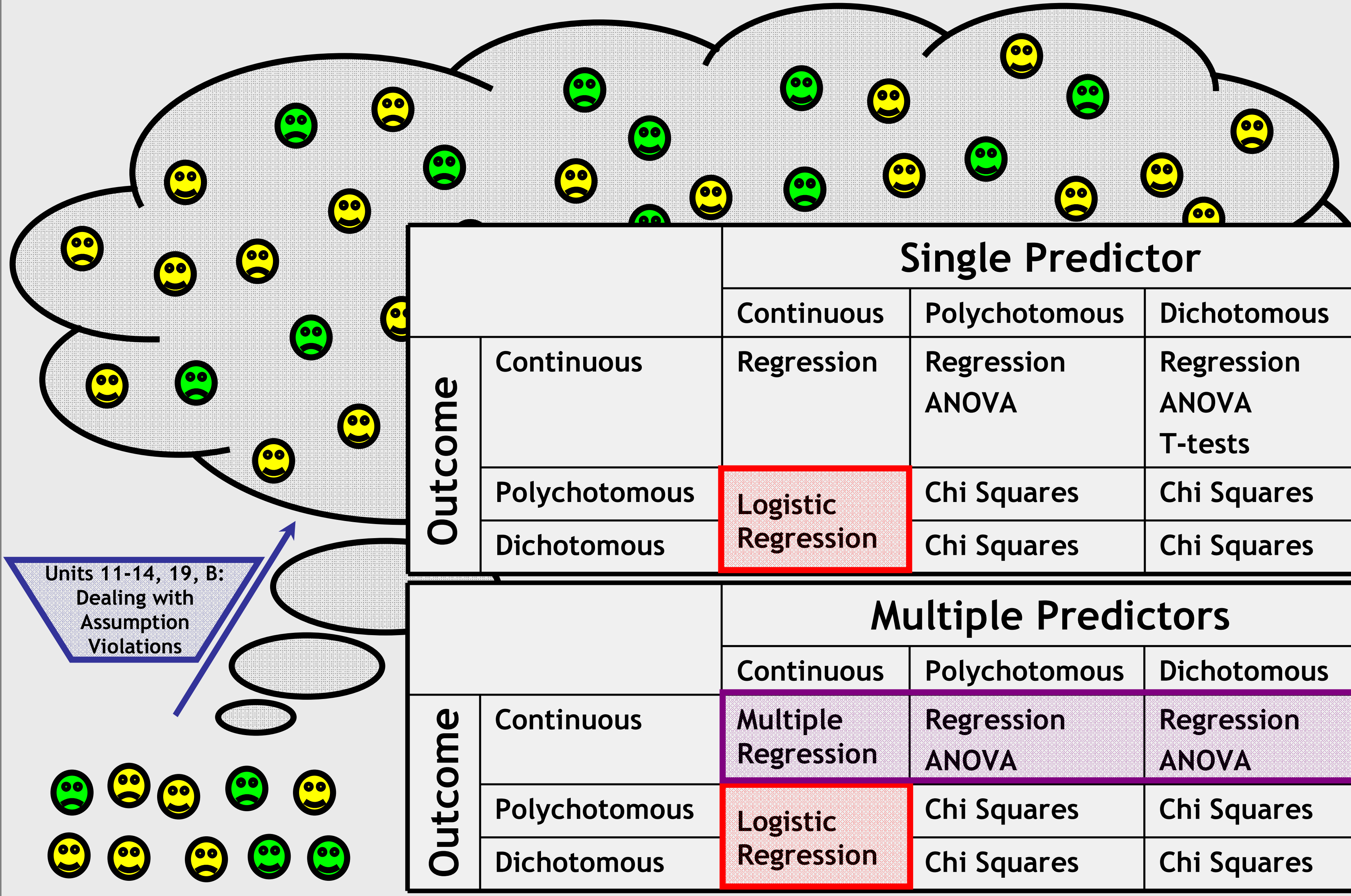
Unit 17: Road Map (VERBAL)

Nationally Representative Sample of 7,800 8th Graders Surveyed in 1988 (NELS 88).
Outcome Variable (aka Dependent Variable):
   READING, a continuous variable, test score, mean = 47 and standard deviation = 9
Predictor Variables (aka Independent Variables):
<u>Question Predictor-</u>
   RACE, a polychotomous variable, 1 = Asian, 2 = Latino, 3 = Black and 4 = White
<u>Control Predictors-</u>
   HOMEWORK, hours per week, a continuous variable, mean = 6.0 and standard deviation = 4.7
   FREELUNCH, a proxy for SES, a dichotomous variable, 1 = Eligible for Free/Reduced Lunch and 0 = Not
   ESL, English as a second language, a dichotomous variable, 1 = ESL, 0 = native speaker of English

- Unit 11: What is measurement error, and how does it affect our analyses?
- Unit 12: What tools can we use to detect assumption violations (e.g., ***<u>outliers</u>***)?
- Unit 13: How do we deal with violations of the ***<u>linearity</u>*** and ***<u>normality</u>*** assumptions?
- Unit 14: How do we deal with violations of the ***<u>homoskedasticity</u>*** assumption?
- Unit 15: What are the correlations among reading, race, ESL, and homework, controlling for SES?
- Unit 16: Is there a relationship between reading and race, controlling for SES, ESL and homework?
- Unit 17: Does the relationship between reading and race vary by levels of SES, ESL or homework?
- Unit 18: What are sensible strategies for building complex statistical models from scratch?
- Unit 19: How do we deal with violations of the ***<u>independence</u>*** assumption (using ANOVA)?

Unit 17: Road Map (Schematic)



Single Predictor

Outcome	Continuous	Polychotomous	Dichotomous
Continuous	Regression	Regression ANOVA	Regression ANOVA T-tests
Polychotomous	Logistic Regression	Chi Squares	Chi Squares
Dichotomous	Logistic Regression	Chi Squares	Chi Squares

Multiple Predictors

Outcome	Continuous	Polychotomous	Dichotomous
Continuous	Multiple Regression	Regression ANOVA	Regression ANOVA
Polychotomous	Logistic Regression	Chi Squares	Chi Squares
Dichotomous	Logistic Regression	Chi Squares	Chi Squares

Unit 17: Roadmap (SPSS Output)

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	48.338	.110		438.242	.000	48.122	48.554
	ASIAN	1.034	.383	.030	2.697	.007	.283	1.786
	BLACK	-4.889	.339	-.161	-14.423	.000	-5.554	-4.225
	LATINO	-4.418	.306	-.161	-14.447	.000	-5.017	-3.818
2	(Constant)	43.878	.280		156.558	.000	43.328	44.427
	ASIAN	.727	.377	.021	1.929	.054	-.012	1.465
	BLACK	-4.796	.333	-.158	-14.412	.000	-5.448	-4.144
	LATINO	-4.123	.301	-.151	-13.715	.000	-4.712	-3.534
	L2HOMEWORKP1	1.766	.102	.188	17.254	.000	1.565	1.967
3	(Constant)	45.381	.284		159.528	.000	44.823	45.938
	ASIAN	.461	.441	.013	1.045	.296	-.404	1.325
	BLACK	-3.622	.331	-.119	-10.956	.000	-4.270	-2.974
	LATINO	-3.311	.366	-.121	-9.035	.000	-4.029	-2.592
	L2HOMEWORKP1	1.603	.100	.170	15.974	.000	1.406	1.799
	ESL	.218	.363	.009	.600	.548	-.494	.930
	FREELUNCH	-3.867	.199	-.213	-19.452	.000	-4.256	-3.477
4	(Constant)	45.358	.288		157.560	.000	44.794	45.923
	ASIAN	-.377	.668	-.011	-.564	.573	-1.687	.933
	BLACK	-3.447	.498	-.113	-6.922	.000	-4.423	-2.471
	LATINO	-2.779	.517	-.102	-5.371	.000	-3.793	-1.765
	L2HOMEWORKP1	1.591	.100	.169	15.866	.000	1.394	1.788
	ESL	-.876	.638	-.035	-1.373	.170	-2.126	.374
	FREELUNCH	-3.574	.235	-.197	-15.208	.000	-4.035	-3.113
	ESLxASIAN	3.245	.999	.080	3.249	.001	1.287	5.202
	ESLxBLACK	5.872	1.885	.036	3.115	.002	2.177	9.568
	ESLxLATINO	.446	.858	.013	.520	.603	-1.235	2.127
	FREELUNCHxASIAN	-2.769	.853	-.041	-3.245	.001	-4.442	-1.096
	FREELUNCHxBLACK	-.751	.666	-.019	-1.127	.260	-2.058	.555
	FREELUNCHxLATINO	-.437	.604	-.012	-.724	.469	-1.622	.747

a. Dependent Variable: READING

Unit 18 · Unit 9 · Unit 16 · Unit 17 · Unit 8 · Unit 11 · Unit 12 · Unit 13 · Unit 14

Unit 17: Statistical Interactions

Unit 17 Post Hole:

Interpret a statistical interaction using spreadsheet software.

Unit 17 Technical Memo and School Board Memo:

Check your final model from Memo 16 for interactions; graph and interpret all statistically significant interactions, but, if none are statistically significant, graph and interpret the interaction with the lowest p-value.

Unit 17 Review:

Review Unit 10.

Unit 17: Technical Memo and School Board Memo

Work Products (Part I of II):

I. Technical Memo: Have one section per analysis. For each section, follow this outline.

   A. Introduction

      i. State a theory (or perhaps hunch) for the relationship—think causally, be creative. (1 Sentence)

      ii. State a research question for each theory (or hunch)—think correlationally, be formal. Now that you know the statistical machinery that justifies an inference from a sample to a population, begin each research question, "In the population,…" (1 Sentence)

      iii. List your variables, and label them "outcome" and "predictor," respectively.

      iv. Include your theoretical model.

   B. Univariate Statistics. Describe your variables, using descriptive statistics. What do they represent or measure?

      i. Describe the data set. (1 Sentence)

      ii. Describe your variables. (1 Paragraph Each)

         a. Define the variable (parenthetically noting the mean and s.d. as descriptive statistics).

         b. Interpret the mean and standard deviation in such a way that your audience begins to form a picture of the way the world is. Never lose sight of the substantive meaning of the numbers.

         c. Polish off the interpretation by discussing whether the mean and standard deviation can be misleading, referencing the median, outliers and/or skew as appropriate.

         d. Note validity threats due to measurement error.

   C. Correlations. Provide an overview of the relationships between your variables using descriptive statistics. Focus first on the relationship between your outcome and question predictor, second-tied on the relationships between your outcome and control predictors, second-tied on the relationships between your question predictor and control predictors, and fourth on the relationship(s) between your control variables.

      a. Include your own simple/partial correlation matrix with a well-written caption.

      b. Interpret your simple correlation matrix. Note what the simple correlation matrix foreshadows for your partial correlation matrix; "cheat" here by peeking at your partial correlation and thinking backwards. Sometimes, your simple correlation matrix reveals possibilities in your partial correlation matrix. Other times, your simple correlation matrix provides foregone conclusions. You can stare at a correlation matrix all day, so limit yourself to two insights.

      c. Interpret your partial correlation matrix controlling for one variable. Note what the partial correlation matrix foreshadows for a partial correlation matrix that controls for two variables. Limit yourself to two insights.

Unit 17: Technical Memo and School Board Memo

Work Products (Part II of II):

I. Technical Memo (continued)

- D. Regression Analysis. Answer your research question using inferential statistics. Weave your strategy into a coherent story.
  - i. Include your fitted model.
  - ii. Use the R^2 statistic to convey the goodness of fit for the model (i.e., strength).
  - iii. To determine statistical significance, test each null hypothesis that the magnitude in the population is zero, reject (or not) the null hypothesis, and draw a conclusion (or not) from the sample to the population.
  - iv. Create, display and discuss a table with a taxonomy of fitted regression models.
  - v. Use spreadsheet software to graph the relationship(s), and include a well-written caption.
  - vi. Describe the direction and magnitude of the relationship(s) in your sample, preferably with illustrative examples. Draw out the substance of your findings through your narrative.
  - vii. Use confidence intervals to describe the precision of your magnitude estimates so that you can discuss the magnitude in the population.
  - viii. If regression diagnostics reveal a problem, describe the problem and the implications for your analysis and, if possible, correct the problem.
    - i. Primarily, check your residual-versus-fitted (RVF) plot. (Glance at the residual histogram and P-P plot.)
    - ii. Check your residual-versus-predictor plots.
    - iii. Check for influential outliers using leverage, residual and influence statistics.
    - iv. Check your main effects assumptions by checking for interactions before you finalize your model.
- X. Exploratory Data Analysis. Explore your data using outlier resistant statistics.
  - i. For each variable, use a coherent narrative to convey the results of your exploratory univariate analysis of the data. Don't lose sight of the substantive meaning of the numbers. (1 Paragraph Each)
    1. Note if the shape foreshadows a need to nonlinearly transform and, if so, which transformation might do the trick.
  - ii. For each relationship between your outcome and predictor, use a coherent narrative to convey the results of your exploratory bivariate analysis of the data. (1 Paragraph Each)
    1. If a relationship is non-linear, transform the outcome and/or predictor to make it linear.
    2. If a relationship is heteroskedastic, consider using robust standard errors.

II. School Board Memo: Concisely, precisely and plainly convey your key findings to a lay audience. Note that, whereas you are building on the technical memo for most of the semester, your school board memo is fresh each week. (Max 200 Words)

III. Memo Metacognitive

Unit 17: Research Question

Theory: Head Start programs provide educationally disadvantaged preschoolers the skills and knowledge to start kindergarten on a level playing field.

Research Question: Controlling for ***SES***, ***ESL*** and ***AGE***, is ***GENERALKNOWLEDGE*** positively correlated with ***HEADSTARTHOURS*** for Latina kindergarteners?

Data Set: ECLS (Early Childhood Longitudinal Study) subset of Latinas with no missing data for the variables below (n = 816)

Variables:

Outcome: (***GENERALKNOWLEDGE***) IRT Scaled Score on a Standardized Test of General Knowledge in Kindergarten

Question Predictor: (***HEADSTARTHOURS***) Hours Per Week of Head Start in the Year Before Kindergarten

Control Predictors:

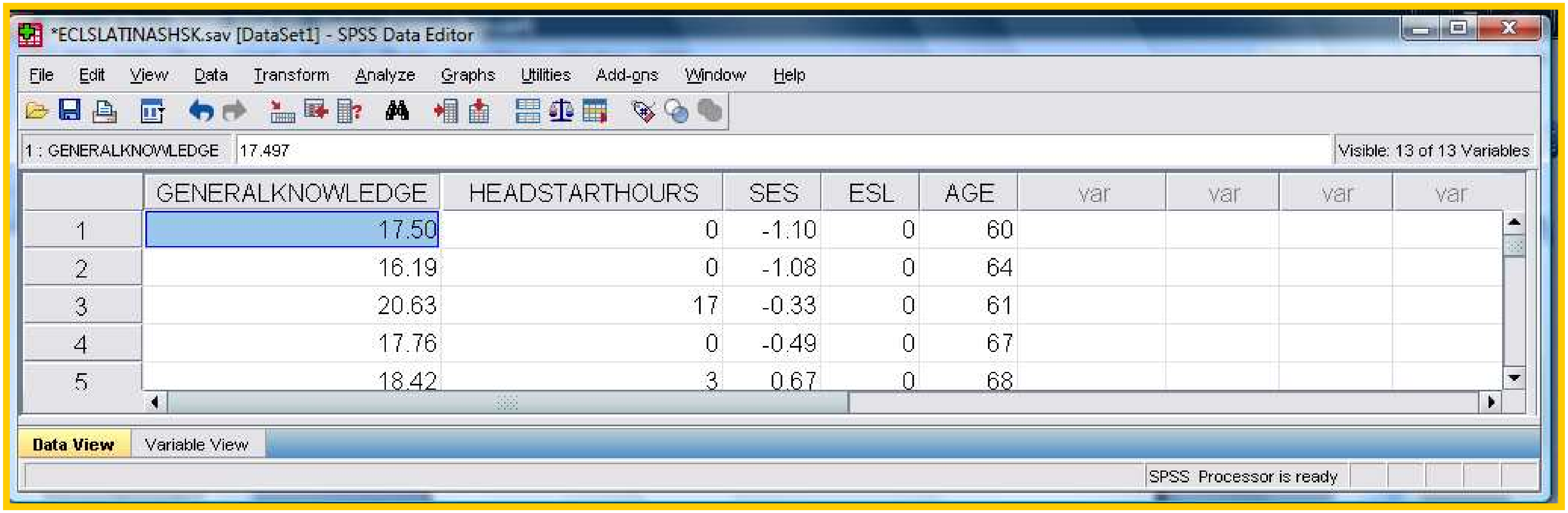
(***SES***) A Composite Measure of the Family's Socioeconomic Status

(***ESL***) A Dichotomy for which 1 Denotes that English is a 2^{nd} Language (0 = Not)

(***AGE***) Age in Months at Kindergarten Entry

Model: $$GENERALKNOWLEDGE = \beta_0 + \beta_1 HEADSTARTHOURS + \beta_2 SES + \beta_3 ESL + \beta_4 AGE + \varepsilon$$

SPSS DATA



*ECLSLATINASHSK.sav [DataSet1] - SPSS Data Editor

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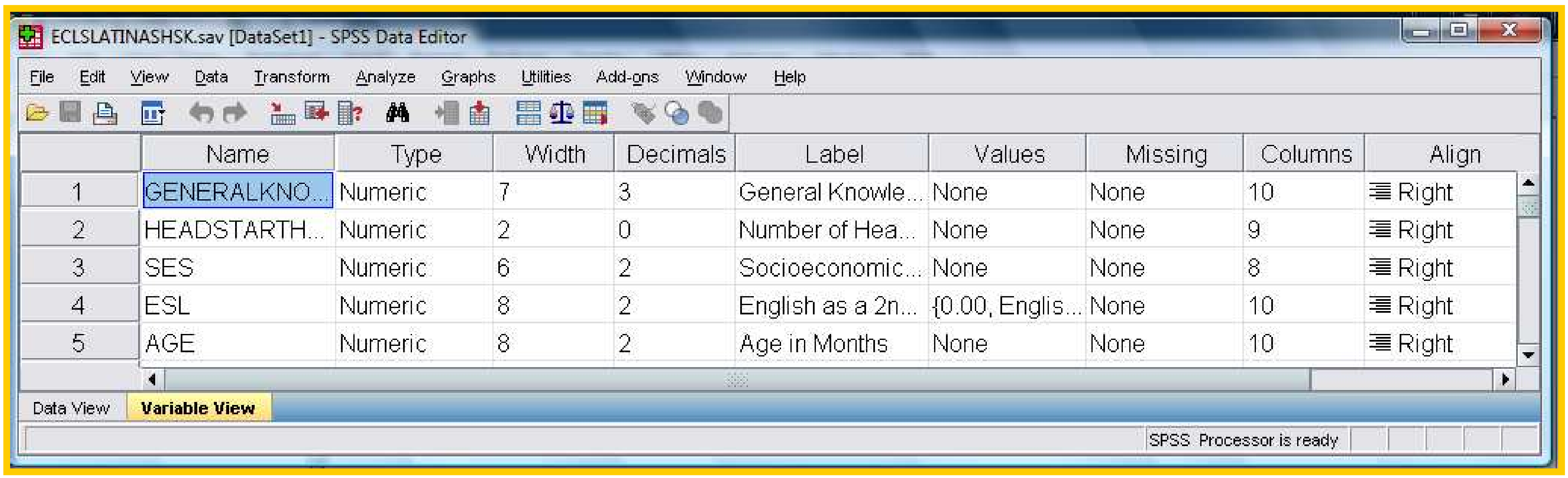
1 : GENERALKNOWLEDGE 17.497 Visible: 13 of 13 Variables

	GENERALKNOWLEDGE	HEADSTARTHOURS	SES	ESL	AGE	var	var	var	var
1	17.50	0	-1.10	0	60				
2	16.19	0	-1.08	0	64				
3	20.63	17	-0.33	0	61				
4	17.76	0	-0.49	0	67				
5	18.42	3	0.67	0	68				

Data View Variable View

SPSS Processor is ready



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	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align
1	GENERALKNO...	Numeric	7	3	General Knowle...	None	None	10	Right
2	HEADSTARTH...	Numeric	2	0	Number of Hea...	None	None	9	Right
3	SES	Numeric	6	2	Socioeconomic...	None	None	8	Right
4	ESL	Numeric	8	2	English as a 2n...	{0.00, Englis...	None	10	Right
5	AGE	Numeric	8	2	Age in Months	None	None	10	Right

Data View Variable View

SPSS Processor is ready

SPSS DATA (With Some "Cross Products" or "Interaction Terms")

*ECLSLATINASHSK.sav [DataSet1] - SPSS Data Editor

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1 : GENERALKNOWLEDGE 17.497 Visible: 13 of 13 Variables

	GENERALKNOWLEDGE	HEADSTARTHOURS	SES	ESL	AGE	ESLxSES	AGESQ	HSHxSES	HSHxESL	HSHxESLxSES	HSHSQ	ESl
1	17.50	0	-1.10	0	60	0	3600	0	0	0	0	
2	16.19	0	-1.08	0	64	0	4096	0	0	0	0	
3	20.63	17	-0.33	0	61	0	3721	-6	0	0	289	
4	17.76	0	-0.49	0	67	0	4489	0	0	0	0	

Data View Variable View

SPSS Processor is ready

ECLSLATINASHSK.sav [DataSet1] - SPSS Data Editor

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	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align
1	GENERALKNO...	Numeric	7	3	General Knowle...	None	None	10	Right
2	HEADSTARTH...	Numeric	2	0	Number of Hea...	None	None	9	Right
3	SES	Numeric	6	2	Socioeconomic...	None	None	8	Right
4	ESL	Numeric	8	2	English as a 2n...	{0.00, Englis...	None	10	Right
5	AGE	Numeric	8	2	Age in Months	None	None	10	Right
6	ESLxSES	Numeric	8	2		None	None	10	Right
7	AGESQ	Numeric	8	2		None	None	10	Right
8	HSHxSES	Numeric	8	2		None	None	10	Right
9	HSHxESL	Numeric	8	2		None	None	10	Right
10	HSHxESLxSES	Numeric	8	2		None	None	13	Right
11	HSHSQ	Numeric	8	2		None	None	10	Right
12	ESLxHSH	Numeric	8	2		None	None	10	Right

Where Last We Left...

$$GENERALKNOWLEDGE = \beta_0 + \beta_1 HEADSTARTHOURS + \beta_2 SES + \beta_3 ESL + \beta_4 AGE + \varepsilon$$

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-3.207	3.247		-.988	.324	-9.580	3.167
	Number of Head Start Hours Per Week	.002	.024	.003	.084	.933	-.045	.049
	Socioeconomic Status Composite Score	4.069	.332	.376	12.240	.000	3.416	4.721
	English as a 2nd Language	-3.784	.460	-.248	-8.217	.000	-4.687	-2.880
	Age in Months	.380	.050	.225	7.655	.000	.283	.478

a. Dependent Variable: General Knowledge IRT Scaled Score

$$GENERAL\hat{K}NOWLEDGE = -3.2 + .002 HEADSTARTHOURS + 4.1 SES - 3.9 ESL + .4 AGE$$

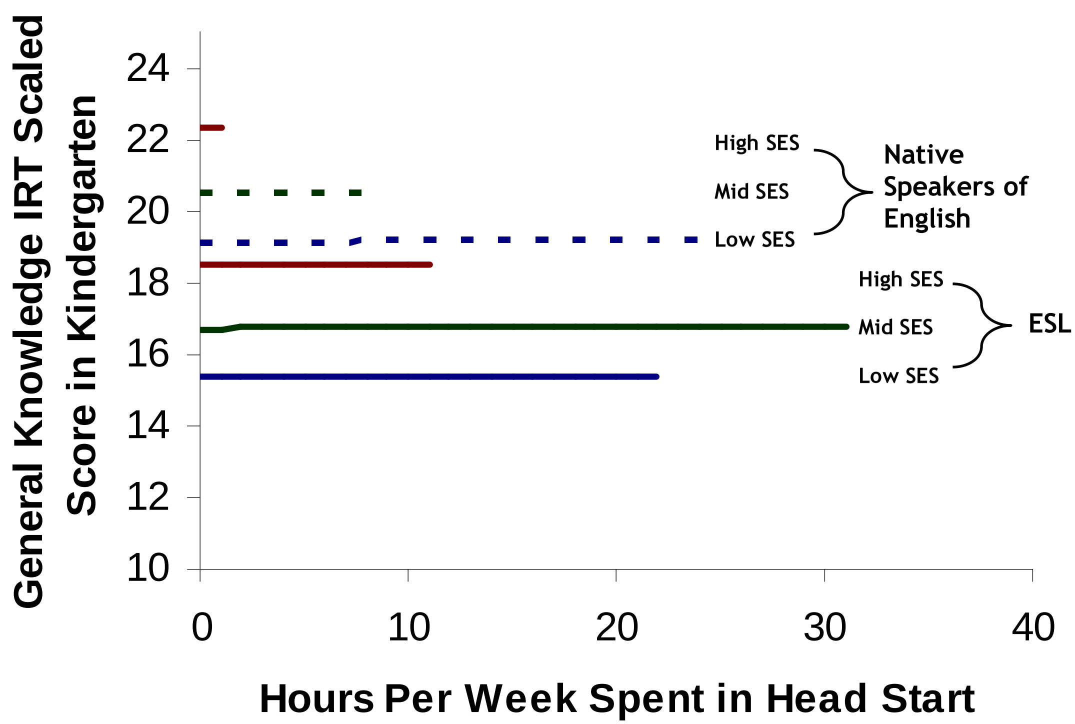
Controlling for SES, ESL, and age, hours of Head Start have a statistically insignificant positive correlation with scores on the kindergarten general knowledge test ($p = .933$). In our sample, when we compare kindergarten Latinas who spent 40 hours per week in Head Start versus kindergarten Latinas who did not attend Head Start, we observe a difference of about .08 points on the general knowledge test favoring Head Starters.

Controlling for Head Start hours, ESL, and age, SES has a statistically significant positive correlation with scores on the kindergarten general knowledge test ($p < .001$). Controlling for Head Start hours, SES, and age, ESL has a statistically significant negative correlation with scores on the kindergarten general knowledge test ($p < .001$). Controlling for Head Start hours, SES, and ESL, age has a statistically significant positive correlation with scores on the kindergarten general knowledge test ($p < .001$).

Note that the constant (i.e., the y-intercept) is our prediction for GENERALKNOWLEDGE when HEADSTARTHOURS is zero, SES is zero, ESL is zero and AGE is zero. Thus, it is meaningless in and of itself, because AGE is never zero.

Graphing A Four-Predictor Model

Figure 16.4. A plot of prototypical fitted values depicting the relationship between HEADSTARTHOURS and GENERALKNOWLEDGE for kindergarten Latinas, controlling for ESL and SES, and holding AGE constant at its median of 64 (n = 816).



Note: I think this is too cluttered, but sometime six lines works.

I chose to hold *AGE* constant at its median:

  50th Percentile of *AGE* = 64

I "chose" prototypical values for *ESL*:

  ESL = 0 and *ESL* = 1

I chose three prototypical values for *SES*:

25th Percentile: -.58
50th Percentile: -.24
75th Percentile: .20

Notice that all our lines have been parallel. That is because we have ***assumed*** them to be parallel—the main effects assumption. In Unit 17, we will relax the main effects assumption when we learn to model statistical interactions.

$$\widehat{GENERALKNOWLEDGE} = -3.2 + .002HEADSTARTHOURS + 4.1SES - 3.8ESL + 0.4AGE$$

$$[\widehat{GENERALKNOWLEDGE} \mid \text{Low SES}\,(SES = -.58), \text{ESL Student}\,(ESL = 1), AGE = 64] =$$
$$-3.2 + .002HEADSTARTHOURS + 4.1*(-.58) - 3.8*(1) + 0.4*(64)$$

Which trend line is associated with this equation?

Relaxing the Main Effects Assumption / Testing for Interactions

Let us test a main effects assumption by asking a supplementary research question:

Does the relationship between *HEADSTARTHOURS* and *GENERALKNOWLEDGE* differ by level of *SES*, controlling for *AGE* and *ESL*, such that the relationship will be greater for students of lower *SES*?

The theory behind the question is that Head Start is designed to help impoverished children, so we do no expect it to be as effective for wealthier children.

Model:

We allow predictors to interact by including cross products in our model.

$$GENERALKNOWLEDGE = \beta_0 + \beta_1 HEADSTARTHOURS + \beta_2 SES + \beta_3 ESL + \beta_4 AGE + \beta_5 HSHxSES + \varepsilon$$

A <u>cross product</u> (or <u>interaction term</u>) is a variable of our creation that allows predictors to "talk with one another" (i.e., statistically interact) when we include it in our model along with the constituent predictors . We call the constituent predictors of a cross product <u>main effects</u>.

SPSS SYNTAX:

Sensibly Named "Cross Product" — Constituent "Main Effect" — Constituent "Main Effect"

```
COMPUTE HSHxSES = HEADSTARTHOURS*SES.
EXECUTE.
```

The cross product is the *product* of multiplying two predictors.

Interpreting Statistical Tests For A Cross Product

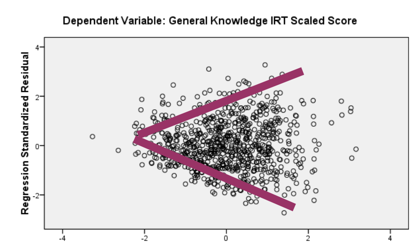
Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	-3.208	3.246		-.988	.323	-9.579	3.163
	Number of Head Start Hours Per Week	-.030	.034	-.038	-.862	.389	-.097	.038
	Socioeconomic Status Composite Score	4.192	.346	.388	12.124	.000	3.513	4.871
	English as a 2nd Language	-3.773	.460	-.247	-8.197	.000	-4.677	-2.870
	Age in Months	.381	.050	.225	7.660	.000	.283	.478
	HSHxSES	-.063	.049	-.058	-1.288	.198	-.159	.033

a. Dependent Variable: General Knowledge IRT Scaled Score

In order to test the main effects assumption that *HEADSTARTHOURS* and *SES* do not interact, we included an interaction term in our model (*HSHxSES*) and tested the null hypothesis that the relationship between *HEADSTARTHOURS* and *GENERALKNOWLEDGE* does not vary by levels of *SES* in the population (when controlling for *ESL* and *AGE*). Based on a p-value of greater than 0.05 (p = .199), we do not reject the null hypothesis, and we continue to make the main effects assumption. Because of heteroskedasticity in our errors, we confirmed our conclusion using robust standard errors (HC3).

We did not "prove" that there is no interaction in the population; i.e., we did not accept the null hypothesis. Rather, we did not find sufficient evidence to overturn the main effects assumption, so we continue to make the main effects assumption.



Rapid Fire Testing

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	-3.182	3.248		-.980	.328	-9.558	3.194
	Number of Head Start Hours Per Week	-.011	.032	-.015	-.357	.721	-.075	.052
	Socioeconomic Status Composite Score	4.055	.333	.375	12.167	.000	3.401	4.709
	English as a 2nd Language	-3.914	.505	-.257	-7.753	.000	-4.905	-2.923
	Age in Months	.381	.050	.225	7.655	.000	.283	.478
	HSHxESL	.030	.047	.027	.631	.528	-.063	.122

a. Dependent Variable: General Knowledge IRT Scaled Score

Nothin'

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	-3.187	3.247		-.982	.327	-9.561	3.187
	Number of Head Start Hours Per Week	-.040	.039	-.050	-1.005	.315	-.117	.038
	Socioeconomic Status Composite Score	4.176	.347	.386	12.025	.000	3.494	4.858
	English as a 2nd Language	-3.881	.505	-.254	-7.680	.000	-4.873	-2.889
	Age in Months	.381	.050	.225	7.659	.000	.283	.478
	HSHxSES	-.061	.049	-.056	-1.237	.217	-.157	.036
	HSHxESL	.025	.047	.022	.519	.604	-.068	.118

a. Dependent Variable: General Knowledge IRT Scaled Score

Testing for a Three-Way Interaction

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-2.976	3.247		-.917	.360	-9.349	3.397
	Number of Head Start Hours Per Week	-.017	.046	-.022	-.372	.710	-.108	.073
	Socioeconomic Status Composite Score	4.592	.400	.425	11.479	.000	3.806	5.377
	English as a 2nd Language	-4.266	.538	-.280	-7.935	.000	-5.321	-3.210
	Age in Months	.378	.050	.224	7.599	.000	.280	.475
	HSHxSES	-.033	.069	-.030	-.475	.635	-.167	.102
	HSHxESL	-.005	.070	-.004	-.065	.948	-.141	.132
	ESLxSES	-1.642	.802	-.087	-2.048	.041	-3.216	-.068
	HSHxESLxSES	-.017	.100	-.012	-.172	.864	-.213	.178

a. Dependent Variable: General Knowledge IRT Scaled Score

In order to test the main effects assumption that *HEADSTARTHOURS* and *SES* and *ESL* do not interact as part of a three-way interaction, we included a three-way interaction term in our model (*HSHxESLxSES*) and tested the null hypothesis that the interaction between *HEADSTARTHOURS* and *SES* does not vary by levels of *ESL* in the population (when controlling for *AGE*). Based on a p-value of greater than 0.05 (p = .864), we do not reject the null hypothesis, and we continue to make the main effects assumption.

Always include the constituent parts of your cross product in your model. For a three-way interaction term, that means including all the constituent interactions. For a two-way interaction term, that means including both main effects. ALWAYS ALWAYS ALWAYS ALWAYS well almost always

COMPUTE HSHxESLxSES = HEADSTARTHOURS*ESL*SES.

Hmm…

What Do We Do Now That We Found an Interaction?

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-2.993	3.238		-.924	.356	-9.348	3.362
	Number of Head Start Hours Per Week	.001	.024	.002	.051	.959	-.046	.048
	Socioeconomic Status Composite Score	4.559	.385	.422	11.839	.000	3.803	5.315
	English as a 2nd Language	-4.301	.504	-.282	-8.541	.000	-5.290	-3.313
	Age in Months	.378	.050	.224	7.624	.000	.280	.475
	ESLxSES	-1.818	.728	-.096	-2.498	.013	-3.246	-.389

a. Dependent Variable: General Knowledge IRT Scaled Score

In order to test the main effects assumption that *SES* and *ESL* do not interact, we included an interaction term in our model (*ESLxSES*) and tested the null hypothesis that the relationship between *GENERALKNOWLEDGE* and *SES* does not vary by levels of *ESL* in the population (when controlling for *AGE* and *HEADSTARTHOURS*). Based on a p-value of less than 0.05 ($p = .013$), we reject the null hypothesis, we reject the main effects assumption, and we conclude that there is an interaction between *ESL* and *SES* in the population. Because of heteroskedasticity in our errors, we confirmed our conclusion using robust standard errors (HC3).

Okay, but what does it mean? How do we interpret the parameter estimate?

Mathematical interpretations can be more confusing than illuminating. Generally, graphical interpretations provide our best hope for understanding the complexity of the relationship. However, because we have an interaction with a dichotomous predictor, the math may be helpful here. We'll start with the math, and then go to the graph, and then stick with graphs.

Using Mathematics to Interpret Interactions

$$\hat{GENERALKNOWLEDGE} = -2.993 + .001HEADSTARTHOURS + 4.559SES - 4.301ESL + .378AGE - 1.818ESLxSES$$

Coef

Model		Unstandardized Coefficients B	Std. Error
1	(Constant)	-2.993	3.238
	Number of Head Start Hours Per Week	.001	.024
	Socioeconomic Status Composite Score	4.559	.385
	English as a 2nd Language	-4.301	.504
	Age in Months	.378	.050
	ESLxSES	-1.818	.728

a. Dependent Variable: General Knowledge IRT Scaled Score

Controlling for hours of Head Start and age, for speakers of English as a native language, a one unit difference in our SES composite is associated with 4.5 point difference on the general knowledge test, but for speakers of English as a second language, a one unit difference in our SES composite is associated with only a 2.7 point difference on the general knowledge test. In short, the relationship between SES and test scores is greater for non-ESL Latinas than ESL Latinas.

Hitherto, we have only changed the y-intercept with our regression decompositions when we substituted a prototypical value for a variable.

For the first time in this course, we are changing a slope when we substitute a prototypical value for a variable.

Speakers of English as a Second Language (ESL = 1):

$$\hat{GENERALKNOWLEDGE} = -2.993 + .001HEADSTARTHOURS + 4.559SES - 4.301(1) + .378AGE - 1.818(1 * SES)$$

$$\hat{GENERALKNOWLEDGE} = -2.993 + .001HEADSTARTHOURS + 4.559SES - 4.301 + .378AGE - 1.818SES$$

$$\hat{GENERALKNOWLEDGE} = -7.294 + .001HEADSTARTHOURS + 2.741SES + .378AGE$$

Combine like terms!

Native Speakers of English (ESL = 0):

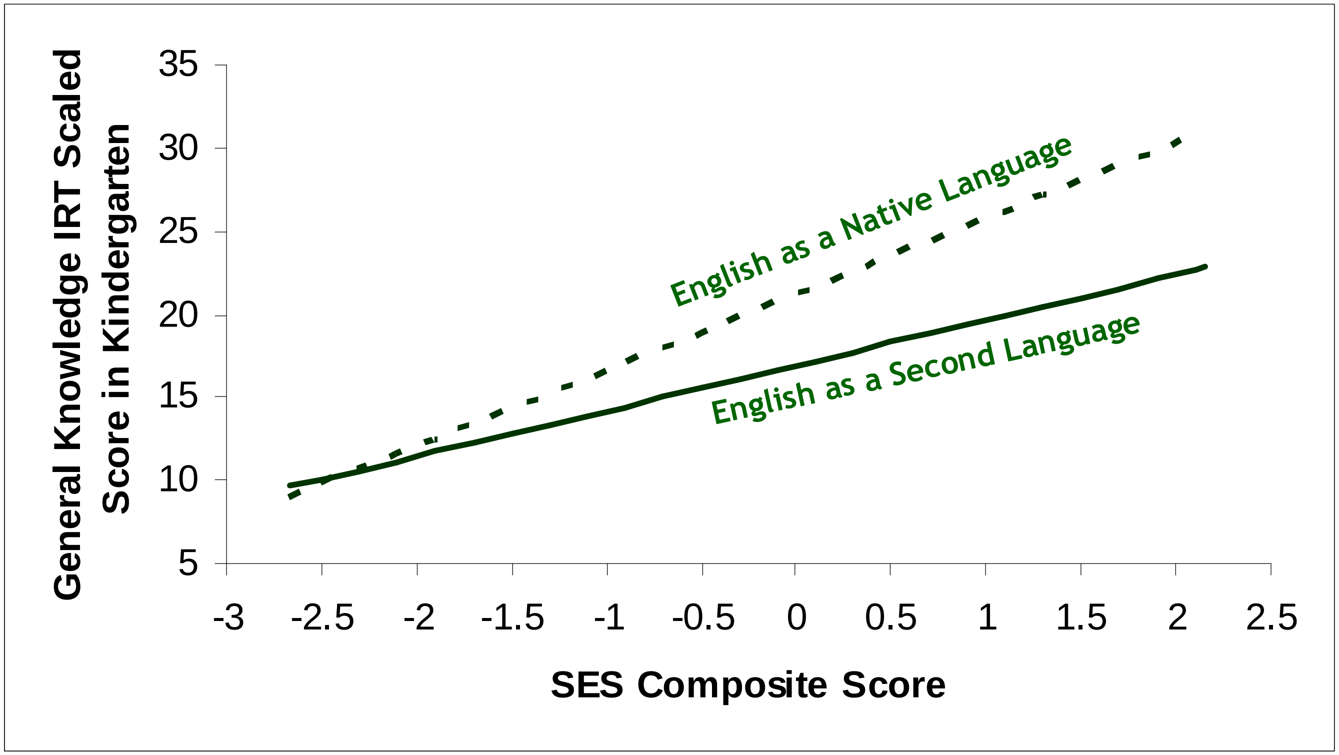
$$\hat{GENERALKNOWLEDGE} = -2.993 + .001HEADSTARTHOURS + 4.559SES - 4.301(0) + .378AGE - 1.818(0 * SES)$$

$$\hat{GENERALKNOWLEDGE} = -2.993 + .001HEADSTARTHOURS + 4.559SES + .378AGE$$

Using Graphs to Interpret Interactions

$$GENERAL\hat{K}NOWLEDGE = -2.993 + .001HEADSTARTHOURS + 4.559SES - 4.301ESL + .378AGE - 1.818ESLxSES$$

Figure 17.1. A plot of prototypical fitted values depicting the statistical interaction between ESL and SES in predicting GENERALKNOWLEDGE, controlling for AGE (held constant at the median, 64 months) and HEADSTARTHOURS (held constant at the mode, 0) (n = 816)



When we compare our predictions for ESL and non-ESL Latinas, our predictions differ little if at all for extremely impoverished students. However, for students within the highest socioeconomic strata, our predictions differ by about 7 points. In general, the ESL gap is related to SES, such that the gap is greater when SES is greater.

Our interaction is about as simple as it gets, but even in this case, the graph communicates instantly what the math fails to communicate in ten minutes of studying.

In order to see the interaction in a graph, one of the main effects should be on the X-axis, and the other main effect must be represented by different lines (i.e., different prototypical values).

If you hold one of the constituent main effects constant, the interaction will not appear at all.

If both main effects get prototypical values, the interaction will appear as different spacings between the parallel lines, which can be hard to see.

Interpreting a Three-Way Interaction (Draw Three Pictures!)

In our sample, we observe a statistically significant three-way interaction between QP, CP1 and CP2 ($p < .05$). ((((If (big IF) you can provide a general characterization of the three-way interaction, do so here.)))) The relationship between O and QP differs by level of CP1. In other words, there is a two-way interaction between QP and C1; however, the two-way interaction itself differs by level of CP2. For the sake of description, we will consider the two-way interaction at three levels of CP2, low, medium, and high.

When CP2 is low, QP and CP1 interact ordinally/disordinally. The relationship between O and QP is positive/negative when CP1 is low, positive/negative when CP1 is medium and negative/positive when CP1 is high. This is expected/unexpected because…((((a few sentences where your goal is rehash the relationships several times in a context that will help them stick to the ribs of the reader))))

When CP2 is medium, QP and CP1 interact ordinally/disordinally. The relationship between O and QP is positive/negative when CP1 is low, positive/negative when CP1 is medium and negative/positive when CP1 is high. This is expected/unexpected because…((((a few sentences where your goal is rehash the relationships several times in a context that will help them stick to the ribs of the reader))))

When CP2 is high, QP and CP1 interact ordinally/disordinally. The relationship between O and QP is positive/negative when CP1 is low, positive/negative when CP1 is medium and negative/positive when CP1 is high. This is expected/unexpected because…((((a few sentences where your goal is rehash the relationships several times in a context that will help them stick to the ribs of the reader))))

Thus, the relationship between O and QP is complex, but we may note some interesting patterns. ((((A few sentences.))))

Understanding Two-Way Interactions:

- Abstractly: Sometimes the relationship between your outcome and your question predictor differs by the level of your control predictor.
- Geometrically: Sometimes your prototypical trend lines are not parallel.
- Practically: Sometimes the effectiveness of your intervention or program differs by gender, SES, age, proficiency etc. (E.g., your program works for girls but not for boys, so you can't just say your program works without qualification.)

Understanding Three-Way Interactions:

- Abstractly: Sometimes a two-way interaction differs by the level of another control predictor.
- Geometrically: Sometimes your non-parallel prototypical trend lines differ in non-parallelness by the levels of another control predictor.
- Practically: Sometimes the effectiveness of your intervention or program differs by gender, SES, age, proficiency etc but that differing itself differs by gender, SES, age, proficiency etc. (E.g., your program works for low SES girls but not low SES boys, but it doesn't works for high SES girls or high SES boys. So, you can't just say your program works better for girls without qualification.)

A Special Kind of Interaction: Quadratic Effects

Some predictors do not interact at all, some predictors interact with other predictors, and some predictors interact with themselves! When a main effect interacts with itself, we call it a quadratic effect, where the relationship between an outcome and predictor varies by levels of that predictor.

Sometime processes feed on themselves. For example, in a world in which it takes money to make money, income can feed upon itself. When a YouTube video goes viral, its popularity snowballs such that, the more popular it is, the more popular it gets.

Does the relationship between *HEADSTARTHOURS* and *GENERALKNOWLEDGE* differ by level of *HEADSTARTHOURS*, controlling for *AGE*, *ESL* and *SES*, such that the relationship will be greater for students with more hours of Head Start?

The theory behind the question is that Head Start requires a critical mass of hours to take full effect so that it can go beyond setting the foundation for basic skills and begin to scale the hierarchy of basic skills.

We allow predictors to interact with themselves by including quadratic terms (cross products with themselves) in our model along with the main effect.

Model:

As with earlier in the unit and as with Unit 13, we are "tricking" our statistical software into thinking that we are doing something easy! To SPSS or R, our cross product is just another variable.

$$GENERALKNOWLEDGE = \beta_0 + \beta_1 HEADSTARTHOURS + \beta_2 SES + \beta_3 ESL + \beta_4 AGE + \beta_5 ESLxSES + \beta_6 HSHSQ + \varepsilon$$

SPSS SYNTAX:

Sensibly Named "Cross Product" | Constituent "Main Effect" | Constituent "Main Effect"

```
COMPUTE HSHSQ = HEADSTARTHOURS*HEADSTARTHOURS.
COMPUTE HSHSQ = HEADSTARTHOURS**2
COMPUTE HSHxHSH = HEADSTARTHOURS**2
```

Instead of multiplying HEADSTARTHOURS by itself, we can simply square it.

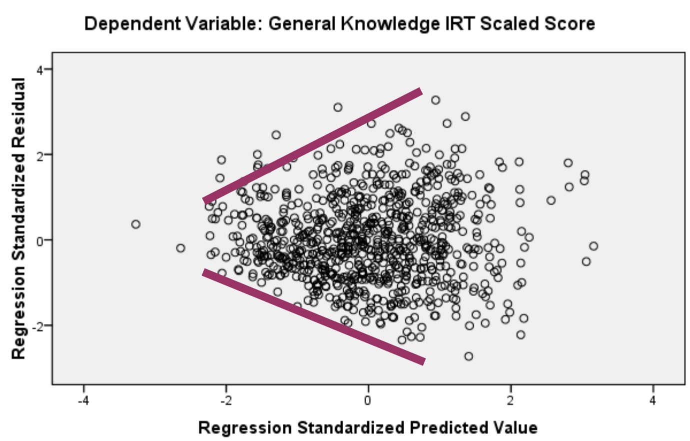
Another sensible name is *HSHxHSH*.

Interpreting A Quadratic Term

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients			95% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	-2.695	3.242		-.831	.406	-9.059	3.668
	Number of Head Start Hours Per Week	-.097	.072	-.123	-1.338	.181	-.239	.045
	Socioeconomic Status Composite Score	4.497	.387	.416	11.615	.000	3.737	5.257
	English as a 2nd Language	-4.291	.503	-.281	-8.526	.000	-5.279	-3.303
	Age in Months	.374	.050	.221	7.539	.000	.277	.471
	ESLxSES	-1.813	.727	-.096	-2.493	.013	-3.241	-.386
	HSHSQ	.003	.002	.130	1.436	.151	-.001	.008

a. Dependent Variable: General Knowledge IRT Scaled Score

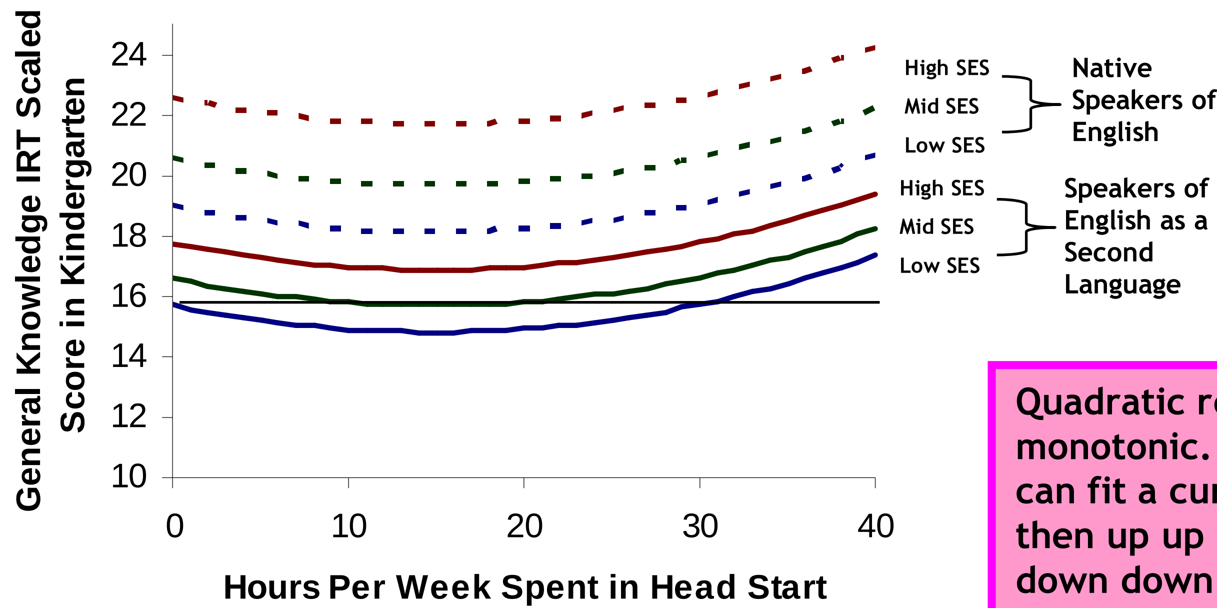


In order to test the main effects assumption that *HEADSTARTHOURS* does not interact with itself, we included a quadratic term in our model (*HSHSQ*) and tested the null hypothesis that the relationship between *HEADSTARTHOURS* and *GENERALKNOWLEDGE* does not vary by levels of *HEADSTARTHOURS* in the population (when controlling for *ESL* and *AGE*). Based on a p-value of greater than 0.05 ($p = .151$), we might be inclined to reject the null hypothesis and consequently continue to make the main effects assumption; however, because of heteroskedasticity in our errors, we used robust standard errors (HC3) and found in fact that the quadratic effect was statistically significant.

Graphic A Quadratic Relationship

$$\widehat{GENERALKNOWLEDGE} = -2.695 - .097HEADSTARTHOURS + 4.497SES - 4.291ESL + .374AGE - 1.813ESLxSES + .003HSHSQ$$

Figure 17.2. A plot of prototypical fitted values depicting the quadratic effect of *HEADSTARTHOURS* in predicting *GENERALKNOWLEDGE*, controlling for AGE (held constant at the median, 64 months) with trend lines for SES and ESL (n = 816)



HSHSQ gets big fast!

$0^2=0,\ 1^2=1,\ 10^2=100,\ 40^2=1600$

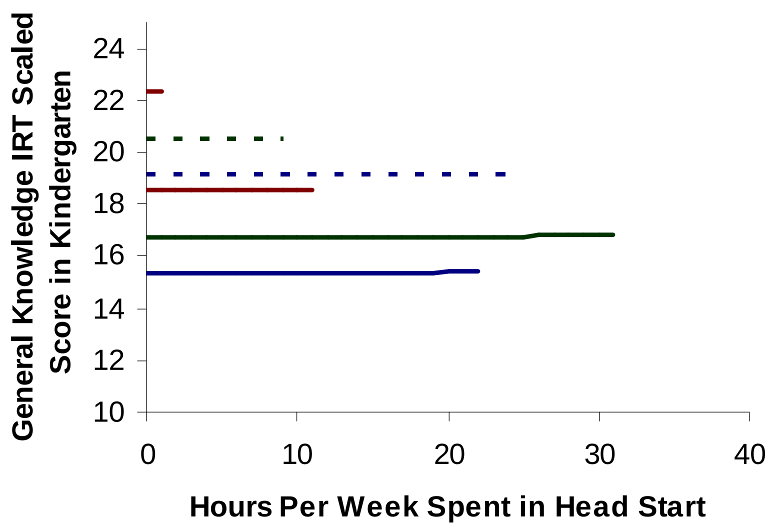
So that "tiny" .003 predicts an additional 5 points on the general knowledge test for students with 40 hours of Head Start per week. You may note that the -.097 coefficient for the main effect "works against" that additional five points, predicting 4 fewer points on the test for the same students.

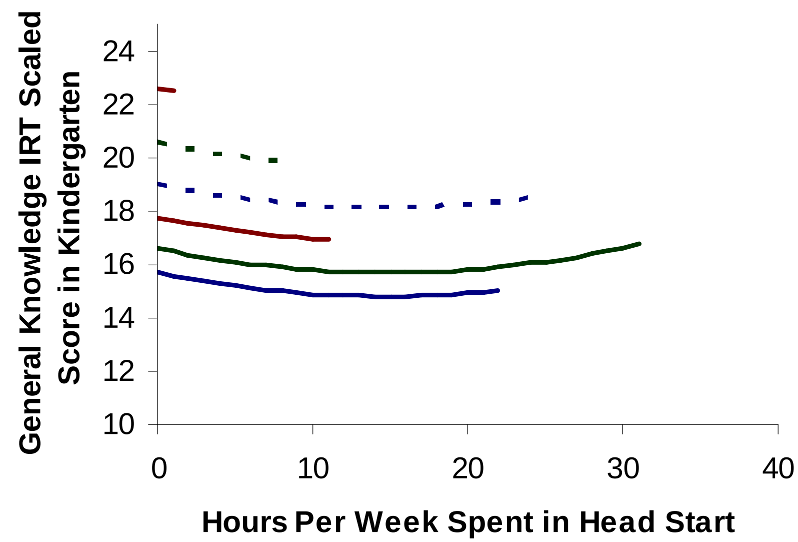
Quadratic relationships can be non-monotonic. For the first time in class, we can fit a curve that goes down down down then up up up (or, up up up then down down down). Thus, we have a new non-linear transformation for our toolbox.

Note that we can see the interaction between ESL and SES by observing the different spacing between the lines. For ESL, the lines are spaced closer together. For non-ESL, the lines are spaced further apart. Contrast that detail to the obviousness of the interaction involving the variable on the X-axis.

Controlling for *SES*, *ESL* and *AGE*, *HEADSTARTHOURS* is negatively correlated with GENERALKNOWLEDGE when hours per week of Head Start are few, but the correlation is positive when hours per week of Head Start exceed thirty.

Quick Comparison To Last Week





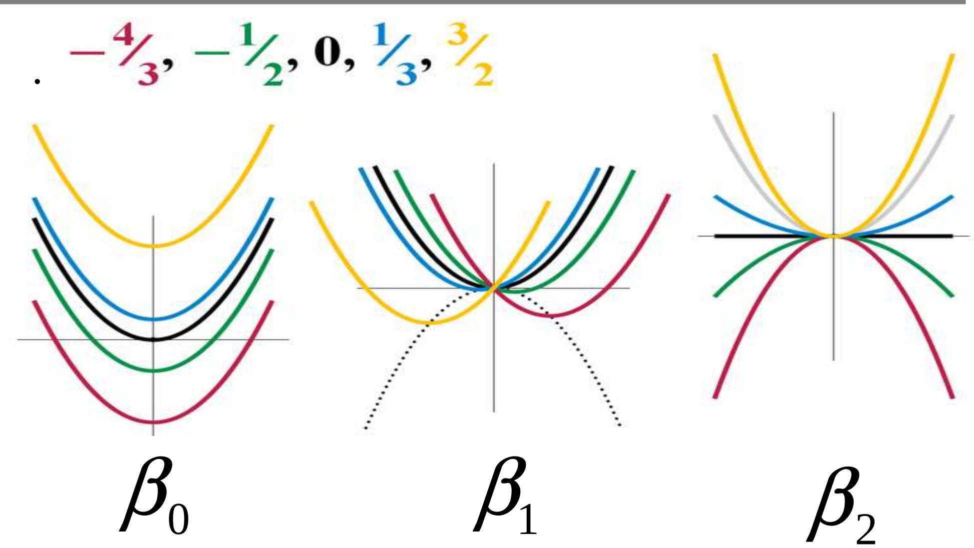
Are we done yet? Not yet. Before I call my model "final," I want to double check to see if my question predictor interacts with any of the other predictors. Lo and behold…

$$y = ax^2 + bx + c$$

$$y = c + bx + ax^2$$

$$GK = \beta_0 + \beta_1 HS + \beta_2 HS^2$$

Note that we learned about quadratics in grade school. Remember those evil quadratic equations that you had to factor. FOIL: first, outer, inner, last. If you were like me, you whined that this had no relevance to your life! Well, as researchers (academic researchers and/or field researchers), we are often interested in quadratic relationships.



http://en.wikipedia.org/wiki/Quadratic_equation

Regression Output for a "Final" Fitted Model

Coefficients[a]

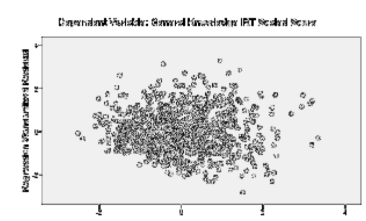
Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	-2.495	3.241		-.770	.442	-8.856	3.867
	Number of Head Start Hours Per Week	-.219	.099	-.279	-2.217	.027	-.414	-.025
	Socioeconomic Status Composite Score	4.429	.393	.410	11.270	.000	3.658	5.201
	English as a 2nd Language	-4.432	.536	-.291	-8.270	.000	-5.484	-3.380
	Age in Months	.371	.050	.220	7.495	.000	.274	.469
	ESLxSES	-1.656	.747	-.087	-2.217	.027	-3.123	-.190
	HSHSQ	.007	.003	.299	2.329	.020	.001	.013
	ESLxHSH	.255	.146	.233	1.752	.080	-.031	.541
	ESLxHSHSQ	-.008	.004	-.247	-1.863	.063	-.017	.000

a. Dependent Variable: General Knowledge IRT Scaled Score

SPSS

GENERALKNO~E	Coef.	Robust HC3 Std. Err.	t	P>|t|	[95% Conf.	Interval]
HEADSTARTH~S	-.2193633	.0806188	-2.72	0.007	-.3776106	-.0611159
SES	4.429404	.4163551	10.64	0.000	3.612138	5.246671
ESL	-4.431899	.5214958	-8.50	0.000	-5.455548	-3.408251
AGE	.3714901	.0511646	7.26	0.000	.2710586	.4719216
ESLXSES	-1.656144	.7381856	-2.24	0.025	-3.105134	-.2071535
HSHSQ	.0073243	.0025661	2.85	0.004	.0022872	.0123614
ESLXHSH	.2550889	.1165213	2.19	0.029	.0263684	.4838095
ESLXHSHSQ	-.0082967	.0036417	-2.28	0.023	-.015445	-.0011483
_cons	-2.494559	3.344323	-0.75	0.456	-9.059158	4.070039

STATA
(with robust standard errors)

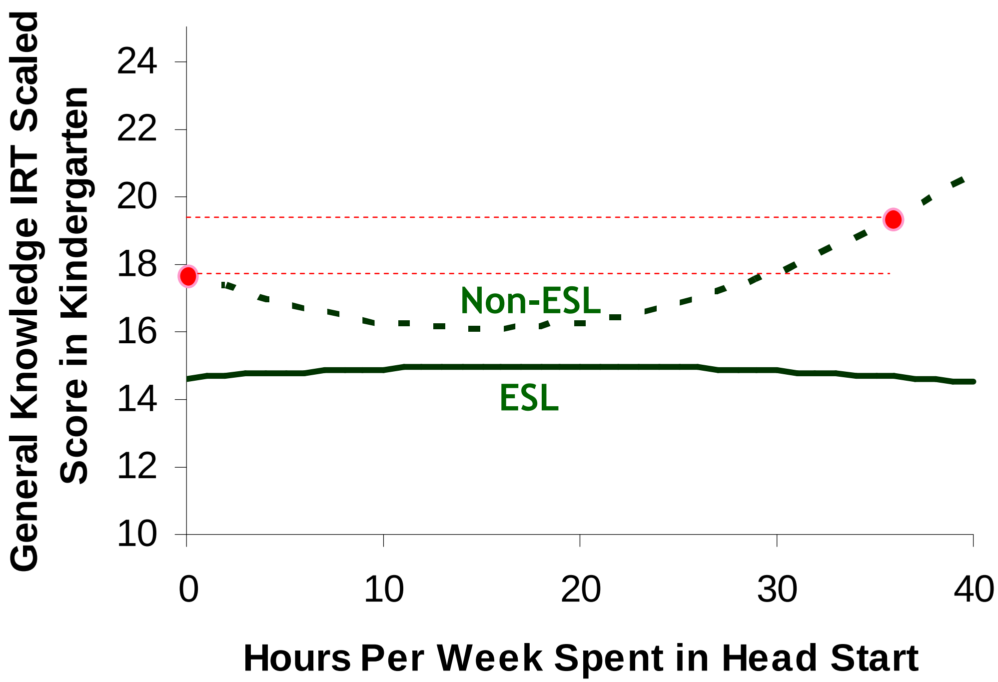


A "Final" Graph

$$GENERALKNOWLEDGE = \beta_0 + \beta_1 HEADSTARTHOURS + \beta_2 SES + \beta_3 ESL + \beta_4 AGE + \beta_5 ESLxSES + \beta_6 HSHSQ + \beta_7 ESLxHSH + \beta_8 ESLxHSHSQ + \varepsilon$$

$$GENERAL\hat{K}NOWLEDGE = -2.5 - .2HEADSTARTHOURS + 4.4SES - 4.4ESL + .4AGE - 1.7ESLxSES + .007HSHSQ + .26ESLxHSH - .008ESLxHSHSQ$$

Figure 17.3. A plot of prototypical fitted values depicting the quadratic effect of *HEADSTARTHOURS* in predicting *GENERALKNOWLEDGE* with trend lines for *ESL*, controlling for *AGE* (held constant at the mean, 64 months) and *SES* (held constant 1 standard deviation below the mean to better represent the Head Start population) (n = 816)



Hold your horses! I note a difference in the graph…

But, not every different we see in our sample is one that we want to infer to the population, i.e., a statistically significant difference. How do we test this?

To be continued in Unit 18…

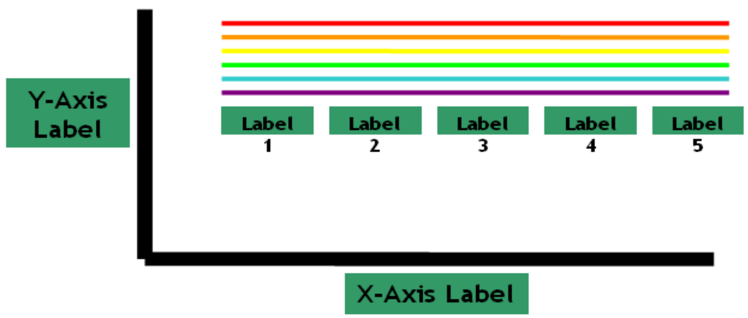
Controlling for *AGE* and *SES*, the relationship between *HEADSTARTHOURS* and *GENERALKNOWLEDGE* differs for ESL Latinas and non-ESL Latinas. For ESL Latinas, there is virtually no relationship. However, for non-ESL Latinas, there is a markedly non-linear relationship such that from 0-20 hours per week, the relationship is negative, but from 20-40 hours per week the relationship is positive. Non-ESL Latinas with over 30 hours per week of Head Start tend to do better on the test than their AGE, SES and non-ESL peers who did not attend Head Start.

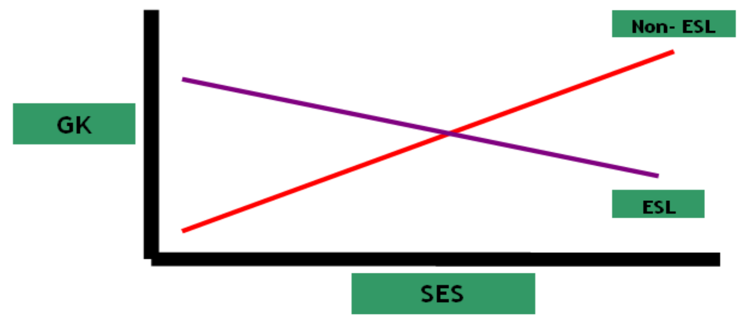
It only makes sense to hold constant a control variable when it does not interact with the question predictor. Once you have an interaction with the question predictor, it is no longer permissible to talk about THE relationship between the outcome and question predictor because there are many relationships depending on the interacting predictor(s).

Dig the Post Hole

Unit 17 Post Hole:

Interpret a statistical interaction using spreadsheet software.





For this post hole, you will not actually use spreadsheet software. Rather, you will do the conceptually hard part of graphing, the preliminary sketch.

1. You will put the predicted outcome on your Y-axis. (This is a given!)
2. What variable will go on your X-axis?
   - This should be a continuous variable.
   - This should be one of your interaction's constituent main effects.
3. For what variable(s) will you choose prototypical values?
   - Generally, you want prototypical values for only one variable.
   - That variable should be the other constituent main effect of your interaction.
   - For categorical predictors, prototypical values are natural. For continuous predictors:
     - ➢ Consider the mean and ±1 standard deviation.
     - ➢ Consider the 25^{th}, 50^{th} and 75^{th} percentiles.
4. What variable(s) will you hold constant at the mean (or median)?
   - Hold constant all predictors that are not part of the interaction.
5. Sketch your graph.
   - Label your axes.
   - Limit the number of lines to the number of prototypical values, and label them.
   - Do NOT spend more than ten seconds trying to get the slopes right. Just make sure they differ!

Interaction FAQs

When do I test for interactions?

1. Test when your theory predicts an interaction. Sometimes your research question involves an interaction. In such cases, be sure to check for the interaction, and include it in your final model even if it is not stat sig. (Your final model should answer your research question even if it does not give you the answer you hypothesized or wanted.) Note that psychologists call interacting control variables “moderators.”
2. Test before you finalize your model. Here you are testing in order to check (and address if necessary) the main effects assumption. Once you have more than a few predictors, the number of possible interactions becomes huge. You cannot (and probably should not) check them all. Check all two-way interactions with your question predictor. Then check any interactions suggested by theory. If you find a statistically significant interaction, temper your enthusiasm at per the next slide.

How do I test for interactions?

1. To test for an interaction in multiple regression, include a cross product in your being sure to also include the constituent main effects.

How do I interpret interactions?

1. Draw the picture!
2. An interaction is when the relationship between the outcome and one predictor varies by the level of another predictor. Focus on the relationship between the outcome and one predictor, and describe it at one level of the interacting predictor, then describe it at another level of the interacting predictor, and then describe it at another level of the interacting predictor. So, let one of the interacting predictors be a “focus predictor” and the other interacting predictor be a “level predictor.” You choose which is which based on your theory/story (the math does not care!).
3. For your super-concise summary fit for an elevator conversation, establish your THREE central variables. You can mention that you are controlling for a slew of other variables, but keep your audience focused on the big THREE. Solidify for them that there is a relationship between your outcome and question predictor. In so doing, build a strong intuition of what the variables mean. Describe a specific relationship, which you know is only associated with a particular level of the interaction, but don't mention (yet) the moderator or moderation (i.e., interacting predictor variable or interaction). Just make them FEEL that particular relationship between your outcome and predictor. Then, pull out the rug... Note that the relationship you just described was for such and such group, but for such and such other group, the relationship is stronger/weaker/negative/positive. Say this last part with a raised eyebrow!

Beware The Interaction Fishing Expedition

One researcher looks for anything and finds something. The other researcher looks for something and finds it. The first researcher is on a data-analytic fishing expedition; she is prone to latching on to any and every spurious correlation that sampling error so plentifully provides. The second researcher, who looks for something (based on her theory), is putting her theory to a meaningful test. If she finds that something, she will support her theory. If not, not. This is why I insist that you incorporate the direction(s) of your hypothesized relationships(s) into your research question.

Sample Size and Interactions (Two Considerations):

1. Meaningful interactions may require a lot of statistical power to detect. Statistical power is what sample size buys you. With small sample sizes, meaningful interactions may be impossible to detect.
2. Be especially thorough with your influence analyses (from Unit 12) when you model statistical interactions. Statistical interactions sometimes permit a few high leverage individuals to exert dictating influence. This is especially true with three-way interactions with small samples.

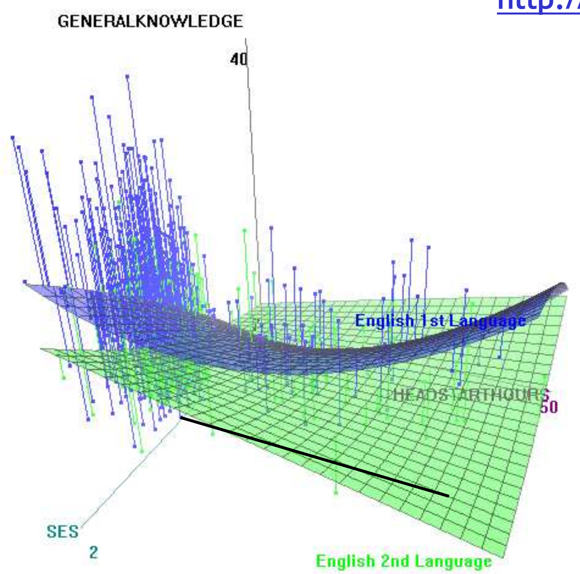
When you are checking for interactions ***for the sake of checking your main effects assumption***, what do you do when you find a statistically significant interaction?

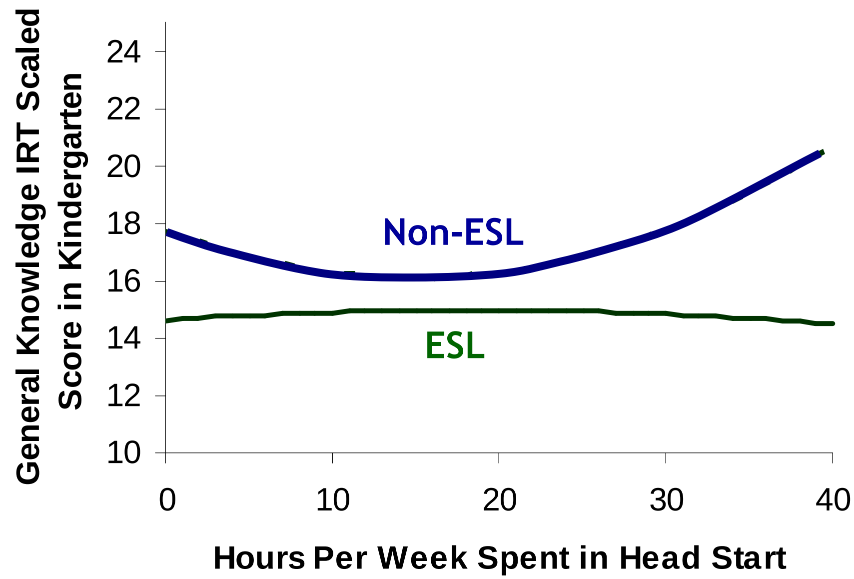
- **WRONG:** Ignore the assumption violation by not modeling the interaction.
- **WRONG:** Report the interaction as though your theory predicted it. (Maybe your theory *could* predict it, but it *did not* predict it, or you would have built the interaction into your research question!)
- **BEST PRACTICE:** Report the interaction as something that turned up in your assumption checking. Suggest that future research confirm that this interaction is not merely an artifact of sampling error.
- **ALTERNATIVE PRACTICE:** Keep a count of all the interactions for which you check. For determining the statistical significance of an interaction, divide your alpha level by the number of checks. This is called a Bonferroni post hoc adjustment. The problem with this practice is that interactions are already difficult to detect, and this will make it much more difficult, so you will be more likely to ignore main effects assumption violations.

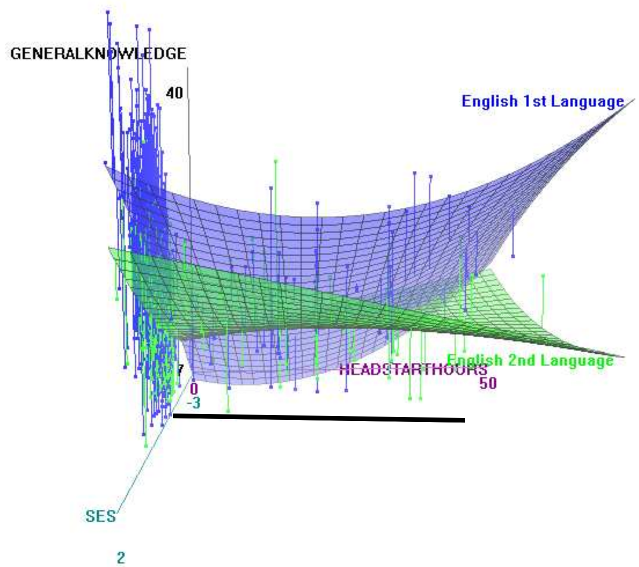
We want to avoid not only violations of the main effect assumption but also Type I Error (i.e., concluding that there is an interaction when it is an artifact of sampling error). But to avoid one is to embrace the other. Scylla and Charybdis.

Interaction in 3-D

http://www.math.yorku.ca/SCS/spida/lm/visreg.html







We can graph the 3-D plot in 2-D if we hold SES constant at 1 standard deviation below the mean. Geometrically, we are graphing a slice of the three dimensional space, slicing at the mean of SES, as depicted by the black lines. Notice that the 2-D slices will differ depending in where we slice SES.

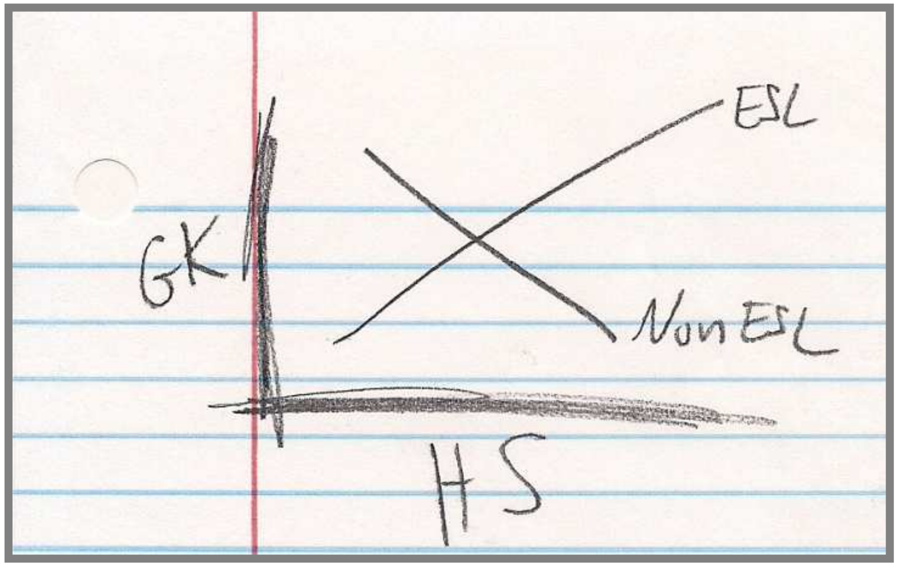
```
library(foreign, pos=4)
ECLS <- read.spss("E:/CD146 2010/Data Sets/ECLS/ECLESHEADSTARTLATINAS.sav",
use.value.labels=TRUE, max.value.labels=Inf, to.data.frame=TRUE)
library(rgl, pos=4)
library(mgcv, pos=4)
scatter3d(ECLS$HEADSTARTHOURS, ECLS$GENERALKNOWLEDGE, ECLS$SES, fit="quadratic",
residuals=TRUE, groups=ECLS$ESL, parallel=FALSE, bg="white", axis.scales=TRUE,
grid=TRUE, ellipsoid=FALSE, xlab="HEADSTARTHOURS", ylab="GENERALKNOWLEDGE",
zlab="SES")
```

Spreadsheet Graphing Nitty Gritty (Part I of II)

Coef

Model		Unstandardized Coefficients B	Std. Error
1	(Constant)	-2.495	3.241
	Number of Head Start Hours Per Week	-.219	.099
	Socioeconomic Status Composite Score	4.429	.393
	English as a 2nd Language	-4.432	.536
	Age in Months	.371	.050
	ESLxSES	-1.656	.747
	HSHSQ	.007	.003
	ESLxHSH	.255	.146
	ESLxHSHSQ	-.008	.004

a. Dependent Variable: General Knowledge IRT Scaled Score



As always, start by sketching the graph and writing down the right-hand side of the fitted model:

= -2.495-.219*(HS)+4.429*(SES)-4.432*(ESL)+.371*(AGE)-1.656*(ESL*SES)+.007*(HS*HS)+.255*(ESL*HS)-.008*(ESL*HS*HS)

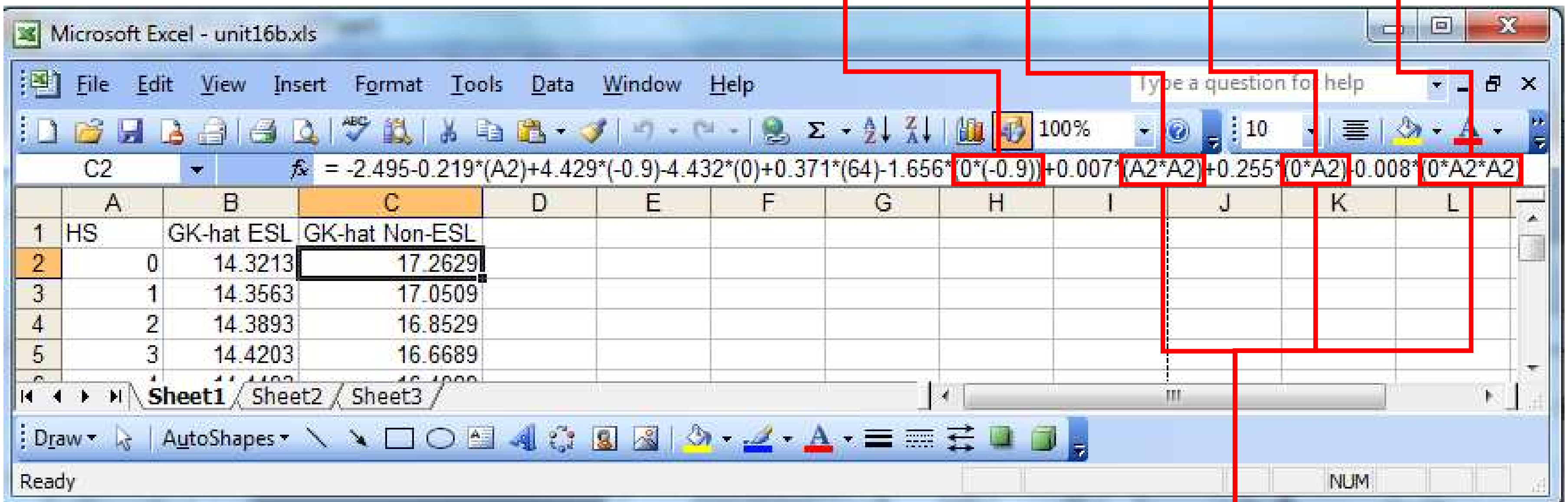
Notice that I "expand" the interaction terms. Our statistical software did not know they were cross products. It thought they were just new variables. But, we know better. Note especially that I expand the square term.

Decisions Decisions:
Predicted-GENERALKNOWLEDGE goes on the y- axis. (This is a no-brainer. It's our outcome.)
HEADSTART goes on the x-axis. (Let's show off our question predictor. The x-axis variable will stand out.)
Two lines (so two prototypical values) for ESL: 0 and 1. (Let's see the interaction with HEADSTART.)
Hold AGE constant at the mean (AGE = 64) and SES constant at -1 standard deviation below the mean (SES = -.9, SES was originally standardized in the full set, but not since it was subsampled to Latinas only).

Spreadsheet Graphing Nitty Gritty (Part II of II)

As always, start by sketching the graph and writing down the right-hand side of the fitted model:

= -2.495-.219*(HS)+4.429*(SES)-4.432*(ESL)+.371*(AGE)-1.656*(ESL*SES)+.007*(HS*HS)+.255*(ESL*HS)-.008*(ESL*HS*HS)



The left column is just a pull down of our x-axis values from the min to the max.

Since we have two lines, we have two columns in addition to the left column.

HS = A2
ESL = 1 for one column
ESL = 0 for another column
SES = (-.9)
AGE = 64

As per our decisions.

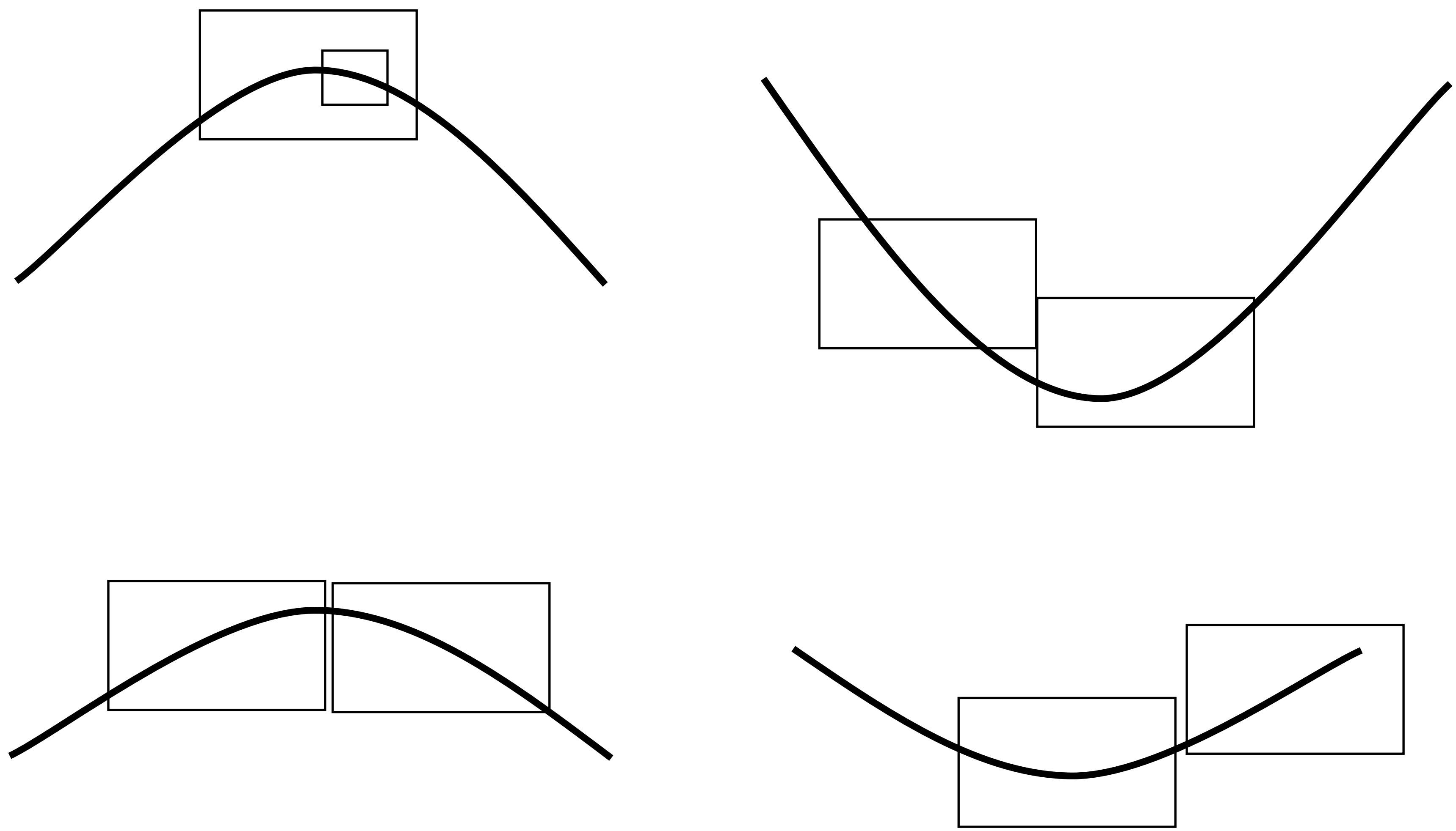
Notice that we replace *all* HS with A2 to pull the value from the left column.

Likewise, other values pop up in various places according to the interactions that we modelled. In fact, AGE is the only non-interacting variable in this model.

We will use a scatterplot once we have filled our columns. Then we tweak.

Quadratic Functions Get One Bend

And, they are symmetric about that bend! But, we don't have to include the whole thing in the range of our data. Often, the bottom (or top) of the bend will be outside our range.



If you need two bends, include a cubic term (VAR^3) for a cubic function.

Unit 17: Roadmap (SPSS Output)

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
4	(Constant)	45.358	.288		157.560	.000	44.794	45.923
	ASIAN	-.377	.668	-.011	-.564	.573	-1.687	.933
	BLACK	-3.447	.498	-.113	-6.922	.000	-4.423	-2.471
	LATINO	-2.779	.517	-.102	-5.371	.000	-3.793	-1.765
	L2HOMEWORKP1	1.591	.100	.169	15.866	.000	1.394	1.788
	ESL	-.876	.638	-.035	-1.373	.170	-2.126	.374
	FREELUNCH	-3.574	.235	-.197	-15.208	.000	-4.035	-3.113
	ESLxASIAN	3.245	.999	.080	3.249	.001	1.287	5.202
	ESLxBLACK	5.872	1.885	.036	3.115	.002	2.177	9.568
	ESLxLATINO	.446	.858	.013	.520	.603	-1.235	2.127
	FREELUNCHxASIAN	-2.769	.853	-.041	-3.245	.001	-4.442	-1.096
	FREELUNCHxBLACK	-.751	.666	-.019	-1.127	.260	-2.058	.555
	FREELUNCHxLATINO	-.437	.604	-.012	-.724	.469	-1.622	.747

a. Dependent Variable: READING

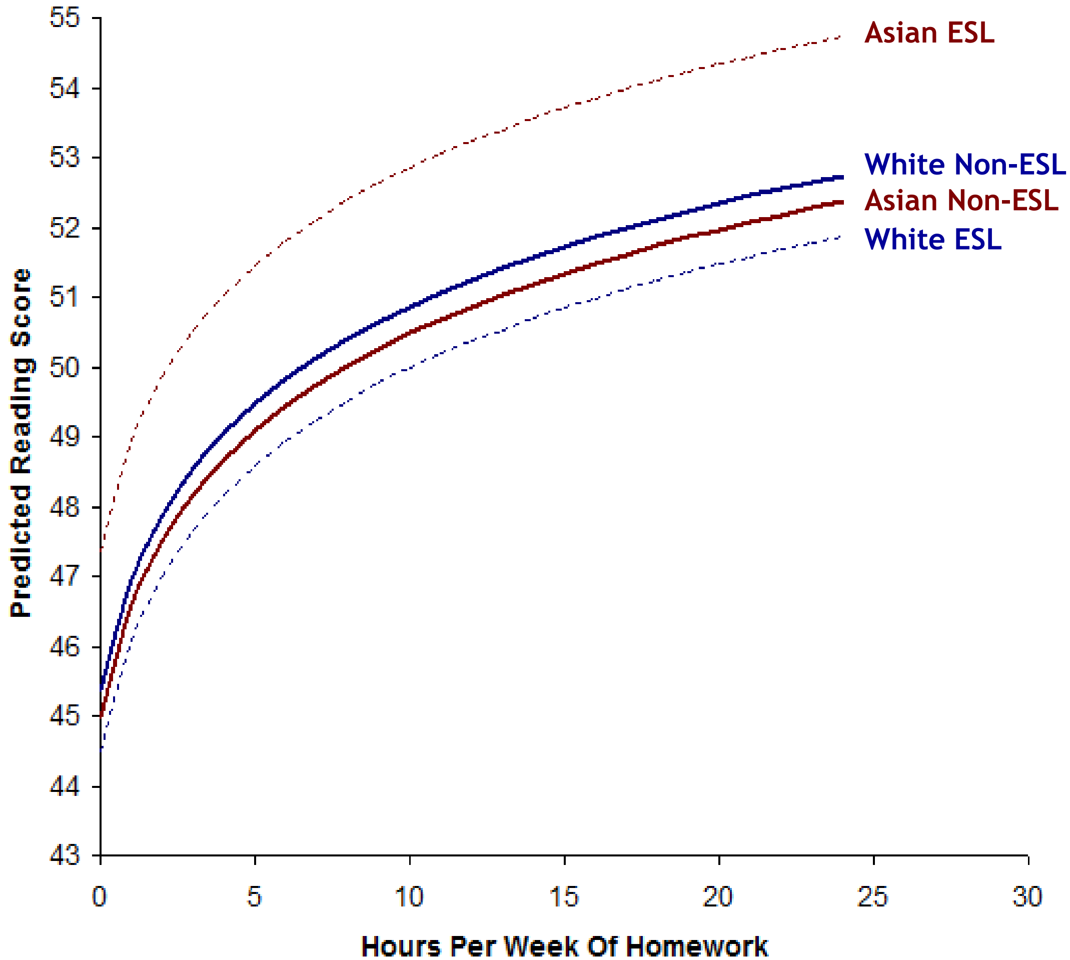
A set of indicator variables (or dummy variables) form a single predictor conceptually. Therefore, if you keep one, you should keep them all. Furthermore, if you interact one, you should interact them all, and if one interaction is statistically significant, then keep all the main effects and cross products.

Although ESLxLATINO, FREELUNCHxBLACK and FREELUNCHxLATINO are not stat. sig., they stay in the model. Why?
Although ESL is not stat. sig., it stays in the model. Why?
Although ASIAN is not stat. sig., it stays in the model. Why? (Two Reasons!)

Answering Our Road Map Question

Unit 17: Does the relationship between reading and race vary by levels of SES, ESL or homework?

Figure 17.X. A plot of prototypical fitted values depicting the relationship between RACE, HOMEWORK, ESL and READING holding FREELUNCH constant at the mode, i.e., not eligible for free lunch, for the White and Asian students in our sample (n = 7,800).



The relationship between reading and race varies by level of SES and ESL. For the sake of illustration, we will compare students who self-identify as White with students who self-identify as Asian, and we will focus on ESL as a moderating variable. Although non-ESL White students and non-ESL Asian students perform statistically indistinguishably, there is a statistically significant difference in reading achievement among ESL White students and ESL Asian students. Whereas ESL White students tend to perform about the same as their non-ESL White counterpart, ESL Asian students tend to read at higher levels of proficiency than their non-ESL Asian counterparts. In fact, controlling for homework hours per week and free lunch eligibility, ESL Asian students read better on average than not only Non-ESL Asian students but also White students in general.

Unit 17 Appendix: Key Concepts

We allow predictors to interact by including cross products in our model.

We did not "prove" that there is no interaction in the population; i.e., we did not accept the null hypothesis. Rather, we did not find sufficient evidence to overturn the main effects assumption, so we continue to make the main effects <u>assumption</u>.

Always include the constituent parts of your cross product in your model. For a three-way interaction term, that means including all the constituent interactions. For a two-way interaction term, that means including both main effects. ALWAYS ALWAYS ALWAYS ALWAYS well almost always

Our interaction is about as simple as it gets, but even in this case, the graph communicates instantly what the math fails to communicate in ten minutes of studying.

- In order to see the interaction in a graph, one of the main effects should be on the X-axis, and the other main effect must be represented by different lines (i.e., different prototypical values).
- If you hold one of the constituent main effects constant, the interaction will not appear at all.
- If both main effects get prototypical values, the interaction will appear as different spacings between the parallel lines, which can be hard to see.

We allow predictors to interact with themselves by including quadratic terms (cross products with themselves) in our model along with the main effect.

Quadratic relationships can be non-monotonic. For the first time in class, we can fit a curve that goes down down down then up up up (or, up up up then down down down). Thus, we have a new non-linear transformation for our toolbox.

It only makes sense to hold constant a control variable when it does not interact with the question predictor. Once you have an interaction with the question predictor, it is no longer permissible to talk about THE relationship between the outcome and question predictor because there are many relationships depending on the interacting predictor(s).

Sample Size and Interactions (Two Considerations):

- Meaningful interactions may require a lot of statistical power to detect. Statistical power is what sample size buys you. With small sample sizes, meaningful interactions may be impossible to detect.
- Be especially thorough with your influence analyses (from Unit 12) when you model statistical interactions. Statistical interactions sometimes permit a few high leverage individuals to exert dictating influence. This is especially true with three-way interactions with small samples.

A set of indicator variables (or dummy variables) form a single predictor conceptually. Therefore, if you keep one, you should keep them all. Furthermore, if you interact one, you should interact them all, and if one interaction is statistically significant, then keep all the main effects and cross products.

Unit 17 Appendix: Key Interpretations

In order to test the main effects assumption that *HEADSTARTHOURS* and *SES* do not interact, we included an interaction term in our model (*HSHxSES*) and tested the null hypothesis that the relationship between *HEADSTARTHOURS* and *GENERALKNOWLEDGE* does not vary by levels of *SES* in the population (when controlling for *ESL* and *AGE*). Based on a p-value of greater than 0.05 ($p = .199$), we do not reject the null hypothesis, and we continue to make the main effects assumption. Because of heteroskedasticity in our errors, we confirmed our conclusion using robust standard errors (HC3).

In order to test the main effects assumption that *HEADSTARTHOURS* and *SES* and *ESL* do not interact as part of a three-way interaction, we included a three-way interaction term in our model (*HSHxESLxSES*) and tested the null hypothesis that the interaction between *HEADSTARTHOURS* and *SES* does not vary by levels of *ESL* in the population (when controlling for *AGE*). Based on a p-value of greater than 0.05 ($p = .864$), we do not reject the null hypothesis, and we continue to make the main effects assumption.

Controlling for hours of Head Start and age, for speakers of English as a native language, a one unit difference in our SES composite is associated with 4.5 point difference on the general knowledge test, but for speakers of English as a second language, a one unit difference in our SES composite is associated with only a 2.7 point difference on the general knowledge test. In short, the relationship between SES and test scores is greater for non-ESL Latinas than ESL Latinas.

When we compare our predictions for ESL and non-ESL Latinas, our predictions differ little if at all for extremely impoverished students. However, for students within the highest socioeconomic strata, our predictions differ by about 7 points. In general, the ESL gap is related to SES, such that the gap is greater when SES is greater.

In order to test the main effects assumption that *HEADSTARTHOURS* does not interact with itself, we included a quadratic term in our model (*HSHSQ*) and tested the null hypothesis that the relationship between *HEADSTARTHOURS* and *GENERALKNOWLEDGE* does not vary by levels of *HEADSTARTHOURS* in the population (when controlling for *ESL* and *AGE*). Based on a p-value of greater than 0.05 ($p = .151$), we might be inclined to reject the null hypothesis and consequently continue to make the main effects assumption; however, because of heteroskedasticity in our errors, we used robust standard errors (HC3) and found in fact that the quadratic effect was statistically significant.

Controlling for *AGE* and *SES*, the relationship between *HEADSTARTHOURS* and *GENERALKNOWLEDGE* differs for ESL Latinas and non-ESL Latinas. For ESL Latinas, there is virtually no relationship. However, for non-ESL Latinas, there is a markedly non-linear relationship such that from 0-20 hours per week, the relationship is negative, but from 20-40 hours per week the relationship is positive. Non-ESL Latinas with over 30 hours per week of Head Start tend to do better on the test than their AGE, SES and non-ESL peers who did not attend Head Start.

Unit 17 Appendix: Key Interpretations

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The relationship between reading and race varies by level of SES and ESL. For the sake of illustration, we will compare students who self-identify as White with students who self-identify as Asian, and we will focus on ESL as a moderating variable. Although non-ESL White students and non-ESL Asian students perform statistically indistinguishably, there is a statistically significant difference in reading achievement among ESL White students and ESL Asian students. Whereas ESL White students tend to perform about the same as their non-ESL White counterpart, ESL Asian students tend to read at higher levels of proficiency than their non-ESL Asian counterparts. In fact, controlling for homework hours per week and free lunch eligibility, ESL Asian students read better on average than not only Non-ESL Asian students but also White students in general.

Three-Way Interactions:

In our sample, we observe a statistically significant three-way interaction between QP, CP1 and CP2 ($p < .05$). ((((If (big IF) you can provide a general characterization of the three-way interaction, do so here.)))) The relationship between O and QP differs by level of CP1. In other words, there is a two-way interaction between QP and C1; however, the two-way interaction itself differs by level of CP2. For the sake of description, we will consider the two-way interaction at three levels of CP2, low, medium, and high.

When CP2 is low, QP and CP1 interact ordinally/disordinally. The relationship between O and QP is positive/negative when CP1 is low, positive/negative when CP1 is medium and negative/positive when CP1 is high. This is expected/unexpected because…((((a few sentences where your goal is rehash the relationships several times in a context that will help them stick to the ribs of the reader))))

When CP2 is medium, QP and CP1 interact ordinally/disordinally. The relationship between O and QP is positive/negative when CP1 is low, positive/negative when CP1 is medium and negative/positive when CP1 is high. This is expected/unexpected because…((((a few sentences where your goal is rehash the relationships several times in a context that will help them stick to the ribs of the reader))))

When CP2 is high, QP and CP1 interact ordinally/disordinally. The relationship between O and QP is positive/negative when CP1 is low, positive/negative when CP1 is medium and negative/positive when CP1 is high. This is expected/unexpected because…((((a few sentences where your goal is rehash the relationships several times in a context that will help them stick to the ribs of the reader))))

Thus, the relationship between O and QP is complex, but we may note some interesting patterns. ((((A few sentences.))))

Unit 17 Appendix: Key Terminology

A cross product (or interaction term) is a variable of our creation that allows predictors to "talk with one another" (i.e., statistically interact) when we include it in our model along with the constituent predictors . We call the constituent predictors of a cross product main effects.

Unit 17 Appendix: Math (Very Optional)

If you want to fit by hand a ***simple*** linear model using ordinary least squares (OLS) regression, you'll need multivariable calculus. Calculus is very good at finding minimums and maximums. When we do OLS regression, we want to find a y-intercept (β_0) and slope (β_1) that minimizes the sum of squared errors (i.e., sum of squared residuals). A statistical error (i.e., residual) is the difference between our observation and prediction. Say that we have three observations:

NAME	*READING*	*FREELUNCH*
Sean	90	0
Betsy	100	0
Waverly	80	1

We propose a model:

$$READING = \beta_0 + \beta_1 FREELUNCH + \varepsilon$$

Thus:

$$READING - \beta_0 - \beta_1 FREELUNCH = \varepsilon$$

Thus:

$$(READING - \beta_0 - \beta_1 FREELUNCH)^2 = (\varepsilon)^2$$

Each subject has a squared error:

$$(90 - \beta_0 - \beta_1 0)^2 = (\varepsilon_{Sean})^2$$

$$(100 - \beta_0 - \beta_1 0)^2 = (\varepsilon_{Betsy})^2$$

$$(80 - \beta_0 - \beta_1 1)^2 = (\varepsilon_{Wavy})^2$$

The sum of squared errors (SSE) is a function of two variables, β_0 and β_1:

$$SSE(\beta_0, \beta_1) = (90 - \beta_0 - \beta_1 0)^2 + (100 - \beta_0 - \beta_1 0)^2 + (80 - \beta_0 - \beta_1 1)^2$$

Unit 17 Appendix: Math (Very Optional)

If you want to fit by hand a multiple linear model ***with interactions*** using ordinary least squares (OLS) regression, you'll follow the same logic as with the simple linear model.

NAME	READING	FREELUNCH	HOMEWORK	FxH
Sean	90	0	0	0
Betsy	100	0	20	0
Waverly	80	1	10	10

We propose a model:

$$READING = \beta_0 + \beta_1 FREELUNCH + \beta_2 HOMEWORK + \beta_3 FxH + \varepsilon$$

Thus:

$$READING - \beta_0 - \beta_1 FREELUNCH - \beta_2 HOMEWORK - \beta_3 FxH = \varepsilon$$

Thus:

$$(READING - \beta_0 - \beta_1 FREELUNCH - \beta_2 HOMEWORK - \beta_3 FxH)^2 = (\varepsilon)^2$$

Each subject has a squared error:

$$(90 - \beta_0 - \beta_1 0 - \beta_2 0 - \beta_2 0)^2 = (\varepsilon_{Sean})^2$$

$$(100 - \beta_0 - \beta_1 0 - \beta_2 20 - \beta_3 0)^2 = (\varepsilon_{Betsy})^2$$

$$(80 - \beta_0 - \beta_1 1 - \beta_2 10 - \beta_3 0)^2 = (\varepsilon_{Wavy})^2$$

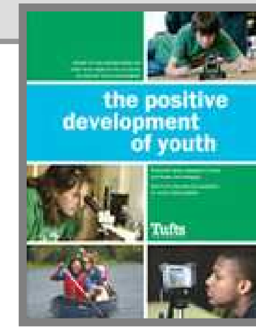
The sum of squared errors (SSE) is a function of FOUR variables, β_0, β_1, β_2 and β_3:

$$SSE(\beta_0, \beta_1, \beta_2, \beta_3) = (90 - \beta_0 - \beta_1 0 - \beta_2 0 - \beta_3 0)^2 + (100 - \beta_0 - \beta_1 0 - \beta_2 20 - \beta_3 0)^2 + (80 - \beta_0 - \beta_1 1 - \beta_2 10 - \beta_3 10)^2$$

Unit 17 Appendix: SPSS Syntax

```
COMPUTE HSHxSES = HEADSTARTHOURS*SES.
EXECUTE.
COMPUTE HSHxESLxSES = HEADSTARTHOURS*ESL*SES.
EXECUTE.
COMPUTE HSHSQ = HEADSTARTHOURS*HEADSTARTHOURS.
COMPUTE HSHSQ = HEADSTARTHOURS**2
COMPUTE HSHxHSH = HEADSTARTHOURS**2
EXECUTE.
```

4-H Study of Positive Youth Development (4H.sav)

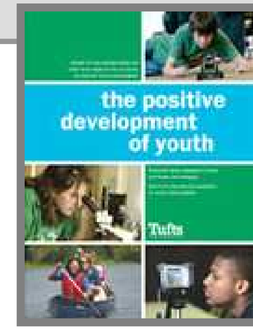


- **4-H Study of Positive Youth Development**
- **Source: Subset of data from IARYD, Tufts University**

- **Sample: These data consist of seventh graders who participated in Wave 3 of the 4-H Study of Positive Youth Development at Tufts University. This subfile is a substantially sampled-down version of the original file, as all the cases with any missing data on these selected variables were eliminated.**
- **Variables:**

(SexFem)	1=Female, 0=Male	(AcadComp)	Self-Perceived Academic Competence
(MothEd)	Years of Mother's Education	(SocComp)	Self-Perceived Social Competence
(Grades)	Self-Reported Grades	(PhysComp)	Self-Perceived Physical Competence
(Depression)	Depression (Continuous)	(PhysApp)	Self-Perceived Physical Appearance
(FrInfl)	Friends' Positive Influences	(CondBeh)	Self-Perceived Conduct Behavior
(PeerSupp)	Peer Support	(SelfWorth)	Self-Worth
(Depressed)	0 = (1-15 on Depression) 1 = Yes (16+ on Depression)		

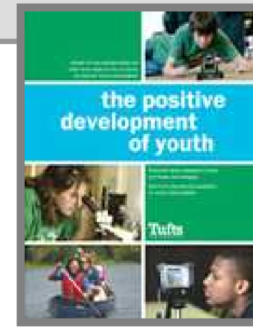
4-H Study of Positive Youth Development (4H.sav)



Statistics

		Grades in School	Female = 1, Male = 0	Self-Worth	Self-Percieved Academic Competence	Birth Mother Education
N	Valid	409	409	409	409	409
	Missing	0	0	0	0	0
Mean		3.3802	.60	3.1209	3.0292	13.86
Std. Deviation		.75184	.491	.60645	.65793	2.289
Minimum		.50	0	1.00	1.00	8
Maximum		4.00	1	4.00	4.00	20
Percentiles	25	3.0000	.00	2.6667	2.5000	12.00
	50	3.5000	1.00	3.1667	3.0000	13.00
	75	4.0000	1.00	3.6667	3.5000	16.00

4-H Study of Positive Youth Development (4H.sav)

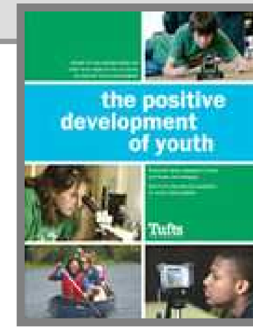


Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	1.342	.315		4.254	.000	.722	1.962
	Female = 1, Male = 0	-.658	.339	-.430	-1.943	.053	-1.324	.008
	Self-Worth	-.083	.091	-.067	-.909	.364	-.263	.097
	Self-Perceived Academic Competence	.556	.057	.486	9.798	.000	.444	.667
	Birth Mother Education	.038	.014	.117	2.718	.007	.011	.066
	SexFemxSelfWorth	.256	.106	.545	2.406	.017	.047	.465

a. Dependent Variable: Grades in School

4-H Study of Positive Youth Development (4H.sav)

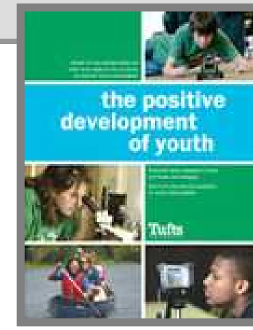


Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-.380	.632		-.602	.548	-1.621	.862
	Female = 1, Male = 0	.153	.063	.100	2.440	.015	.030	.276
	Self-Worth	.486	.202	.392	2.400	.017	.088	.884
	Self-Perceived Academic Competence	.977	.208	.855	4.700	.000	.568	1.385
	Birth Mother Education	.036	.014	.111	2.569	.011	.009	.064
	AcadCompxSelfWorth	-.135	.065	-.607	-2.072	.039	-.264	-.007

a. Dependent Variable: Grades in School

4-H Study of Positive Youth Development (4H.sav)

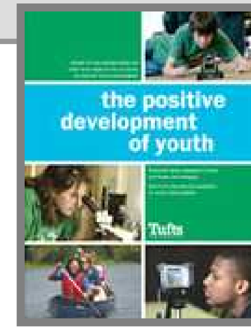


Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-1.018	1.040		-.979	.328	-3.063	1.026
	Female = 1, Male = 0	.137	.063	.089	2.190	.029	.014	.260
	Self-Worth	.677	.329	.546	2.057	.040	.030	1.323
	Self-Perceived Academic Competence	.563	.057	.492	9.900	.000	.451	.674
	Birth Mother Education	.172	.076	.524	2.264	.024	.023	.322
	MothEdxSelfWorth	-.043	.024	-.690	-1.829	.068	-.090	.003

a. Dependent Variable: Grades in School

4-H Study of Positive Youth Development (4H.sav)

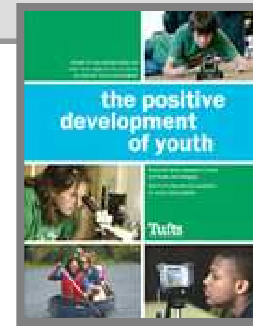


Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-.637	.594		-1.072	.284	-1.804	.531
	Female = 1, Male = 0	.158	.062	.103	2.531	.012	.035	.281
	Self-Worth	1.095	.380	.884	2.886	.004	.349	1.842
	Self-Perceived Academic Competence	.574	.057	.503	10.123	.000	.463	.686
	Birth Mother Education	.035	.014	.107	2.489	.013	.007	.063
	SelfWorthxSelfWorth	-.170	.063	-.827	-2.695	.007	-.295	-.046

a. Dependent Variable: Grades in School

4-H Study of Positive Youth Development (4H.sav)

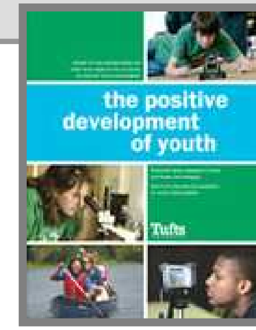


Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-1.366	.581		-2.349	.019	-2.509	-.223
	Female = 1, Male = 0	.172	.062	.112	2.779	.006	.050	.293
	Self-Worth	.085	.058	.069	1.463	.144	-.029	.200
	Self-Perceived Academic Competence	2.097	.376	1.835	5.574	.000	1.357	2.836
	Birth Mother Education	.039	.014	.119	2.807	.005	.012	.066
	AcadCompxAcadComp	-.262	.063	-1.356	-4.125	.000	-.387	-.137

a. Dependent Variable: Grades in School

4-H Study of Positive Youth Development (4H.sav)

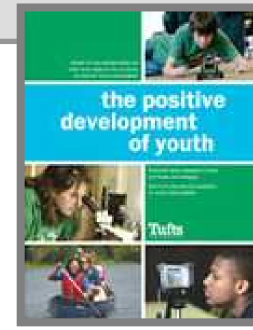


Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	.275	.703		.391	.696	-1.106	1.656
	Female = 1, Male = 0	-.554	.344	-.362	-1.612	.108	-1.229	.122
	Self-Worth	.269	.227	.217	1.188	.236	-.176	.715
	Self-Perceived Academic Competence	.900	.210	.787	4.280	.000	.486	1.313
	Birth Mother Education	.039	.014	.118	2.746	.006	.011	.067
	AcadCompxSelfWorth	-.112	.066	-.502	-1.698	.090	-.242	.018
	SexFemxSelfWorth	.225	.108	.480	2.091	.037	.013	.437

a. Dependent Variable: Grades in School

4-H Study of Positive Youth Development (4H.sav)

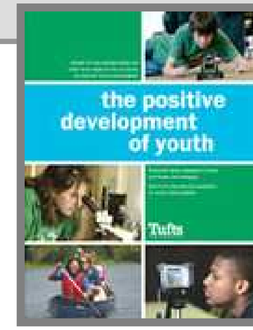


Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients			95% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	-.831	.654		-1.271	.204	-2.117	.454
	Female = 1, Male = 0	-.418	.339	-.273	-1.232	.219	-1.085	.249
	Self-Worth	-.038	.091	-.031	-.417	.677	-.216	.141
	Self-Perceived Academic Competence	1.979	.381	1.731	5.192	.000	1.230	2.728
	Birth Mother Education	.041	.014	.125	2.943	.003	.014	.068
	SexFemxSelfWorth	.188	.106	.400	1.767	.078	-.021	.397
	AcadCompxAcadComp	-.243	.064	-1.256	-3.774	.000	-.369	-.116

a. Dependent Variable: Grades in School

4-H Study of Positive Youth Development (4H.sav)



Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-2.620	.948		-2.763	.006	-4.484	-.756
	Female = 1, Male = 0	.176	.062	.115	2.844	.005	.054	.297
	Self-Worth	.088	.058	.071	1.505	.133	-.027	.202
	Self-Perceived Academic Competence	2.043	.377	1.787	5.422	.000	1.302	2.783
	Birth Mother Education	.230	.115	.700	2.000	.046	.004	.456
	AcadCompxAcadComp	-.254	.064	-1.313	-3.993	.000	-.379	-.129
	MothEdxMothEd	-.007	.004	-.584	-1.672	.095	-.015	.001

a. Dependent Variable: Grades in School

Table 1. Parameter estimates, (standard errors), approximate p values and goodness-of-fit tests for a nested taxonomy of regression models that describe the relationship between perceived violence and concentrated disadvantage controlling for collective efficacy in Chicago neighborhoods (n=342)

	Models				
	M1	**M2**	**M3**	**M4**	**M5**
Intercept	28.2*** (0.7)				
Concentrated Disadvantage	1.6* (0.7)				
Collective Efficacy					
Disadvan/Efficacy Interaction					
Disadvan/Disadvan Interaction					
R^2	0.01*				
df(Residual)	340				
$\bar{D}R^2$					
df ($\bar{D}R^2$)					

Key: ~p <.10 *p<.05; **p<.01; ***p<.001

$$Per\hat{c}Viol = 28.2 + 1.6 ZDisadv$$

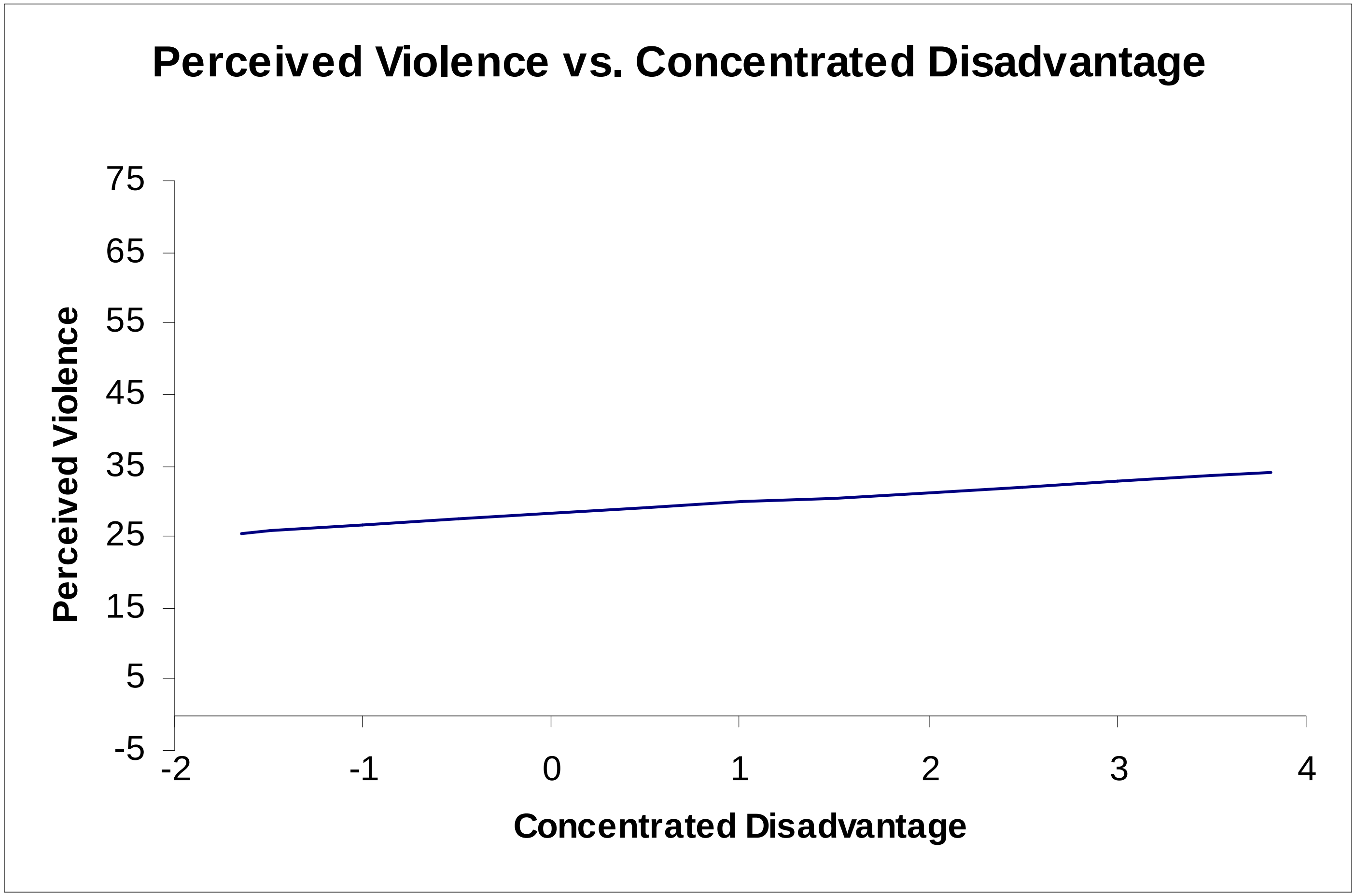


Table 1. **Parameter estimates, (standard errors), approximate p values and goodness-of-fit tests for a nested taxonomy of regression models that describe the relationship between perceived violence and concentrated disadvantage controlling for collective efficacy in Chicago neighborhoods (n=342)**

	Models	
	M1	**M2**
Intercept	28.2*** (0.7)	28.2*** (0.7)
Concentrated Disadvantage	1.6* (0.7)	-1.8* (0.9)
Collective Efficacy		-5.4*** (0.9)
Disadvan/Efficacy Interaction		
Disadvan/Disadvan Interaction		
R^2	0.01*	0.11***
df(Residual)	340	339
$\bar{D}R^2$		0.10***
df ($\bar{D}R^2$)		1

Key: ~p <.10 *p<.05; **p<.01; ***p<.001

$$Per\hat{c}Viol = 28.2 - 1.8ZDisadv - 5.4ZCollEff$$

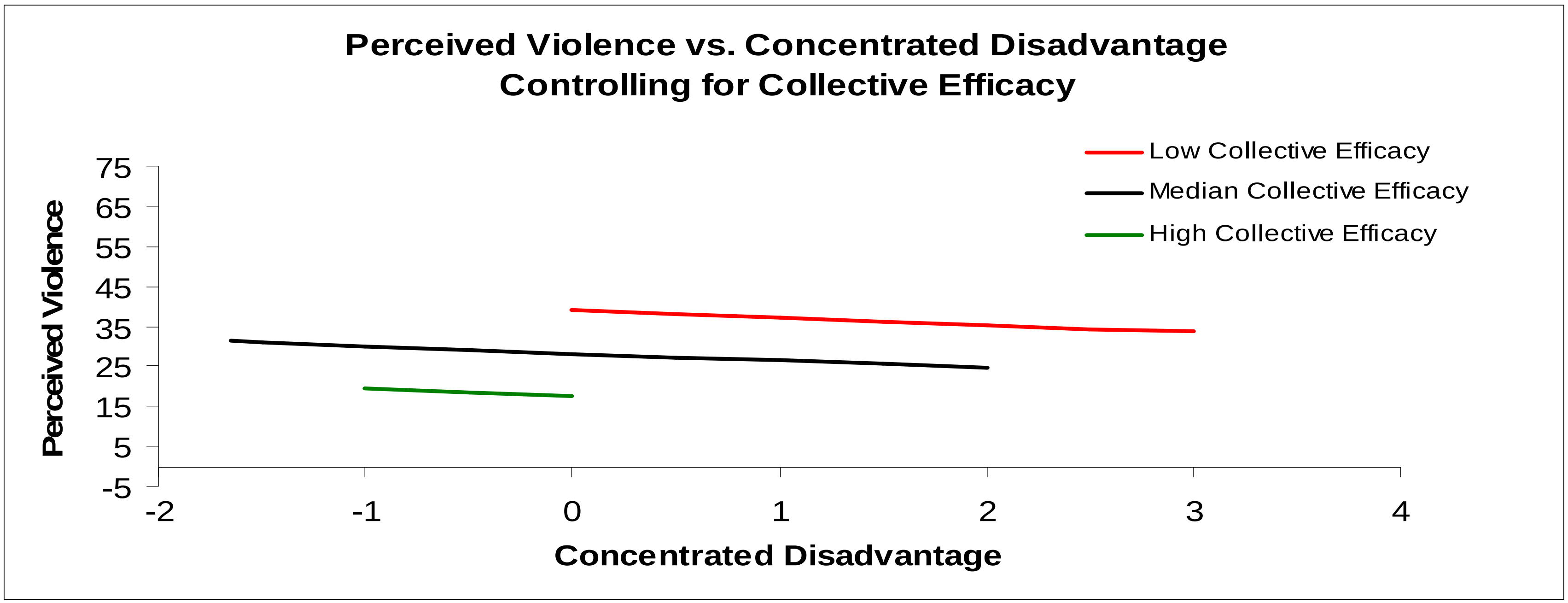


Table 1. **Parameter estimates, (standard errors), approximate p values and goodness-of-fit tests for a nested taxonomy of regression models that describe the relationship between perceived violence and concentrated disadvantage controlling for collective efficacy in Chicago neighborhoods (n=342)**

	Models		
	M1	**M2**	**M3**
Intercept	28.2*** (0.7)	28.2*** (0.7)	29.1*** (0.8)
Concentrated Disadvantage	1.6* (0.7)	-1.8* (0.9)	-0.9 (1.0)
Collective Efficacy		-5.4*** (0.9)	-4.9*** (0.9)
Disadvan/Efficacy Interaction			1.4~ (0.7)
Disadvan/Disadvan Interaction			
R^2	0.01*	0.11***	0.12***
df(Residual)	340	339	338
$\bar{D}R^2$		0.10***	0.01~
df ($\bar{D}R^2$)		1	1

Key: ~p <.10 *p<.05; **p<.01; ***p<.001

$$\hat{PercViol} = 29.1 - 1.0ZDisadv - 4.8ZCollEff + 1.4ZDisadv * ZCollEff$$

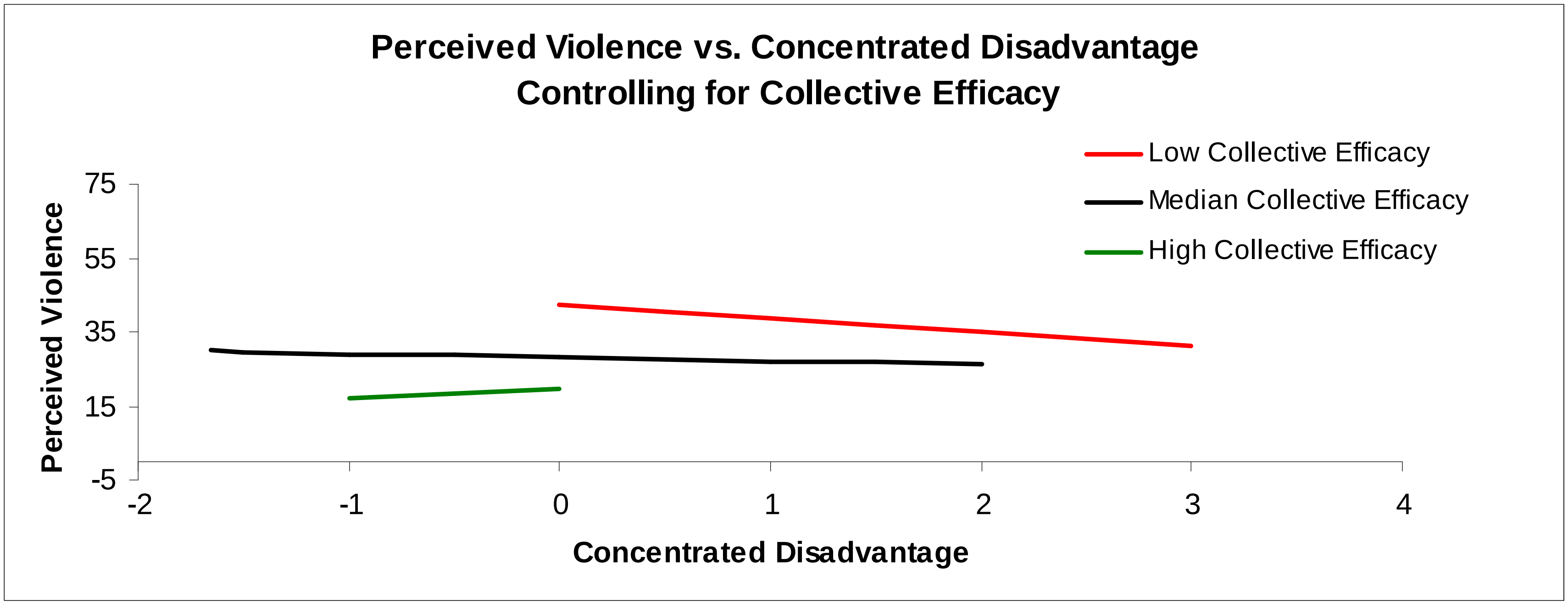
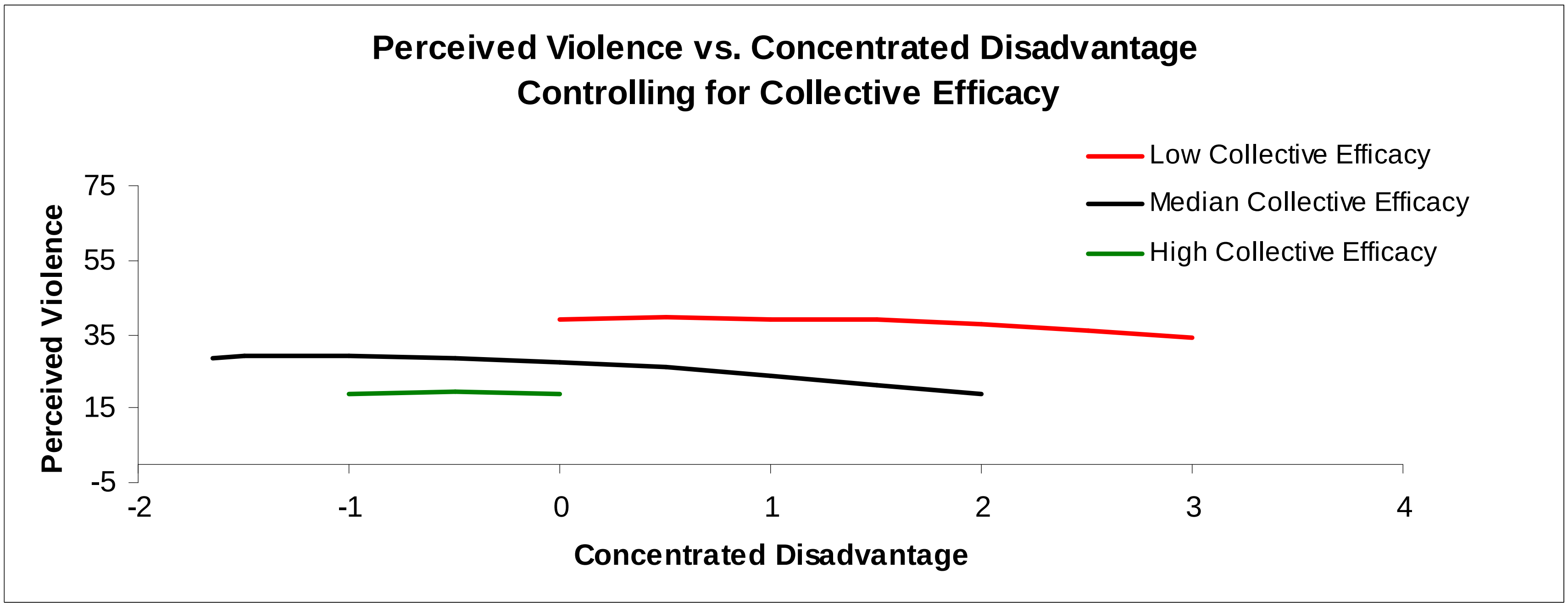


Table 1. **Parameter estimates, (standard errors), approximate p values and goodness-of-fit tests for a nested taxonomy of regression models that describe the relationship between perceived violence and concentrated disadvantage controlling for collective efficacy in Chicago neighborhoods (n=342)**

	Models			
	M1	**M2**	**M3**	**M4**
Intercept	28.2*** (0.7)	28.2*** (0.7)	29.1*** (0.8)	29.2*** (0.8)
Concentrated Disadvantage	1.6* (0.7)	-1.8* (0.9)	-0.9 (1.0)	-0.7 (1.1)
Collective Efficacy		-5.4*** (0.9)	-4.9*** (0.9)	-5.1*** (0.9)
Disadvan/Efficacy Interaction			1.4~ (0.7)	(Note that comparison statistics are with M2.)
Disadvan/Disadvan Interaction				-1.0~ (0.5)
R^2	0.01*	0.11***	0.12***	0.12***
df(Residual)	340	339	338	338
$\bar{D}R^2$		0.10***	0.01~	0.01~
df ($\bar{D}R^2$)		1	1	1

Key: ~p <.10 *p<.05; **p<.01; ***p<.001

$$Per\hat{c}Viol = 29.2 - 0.7ZDisadv - 5.1ZCollEff \; - 1.0ZDisadv * ZDisadv$$



$$\hat{PercViol} = 29.7 + 3.0ZDisadv - 1.5ZDisadvSQ$$

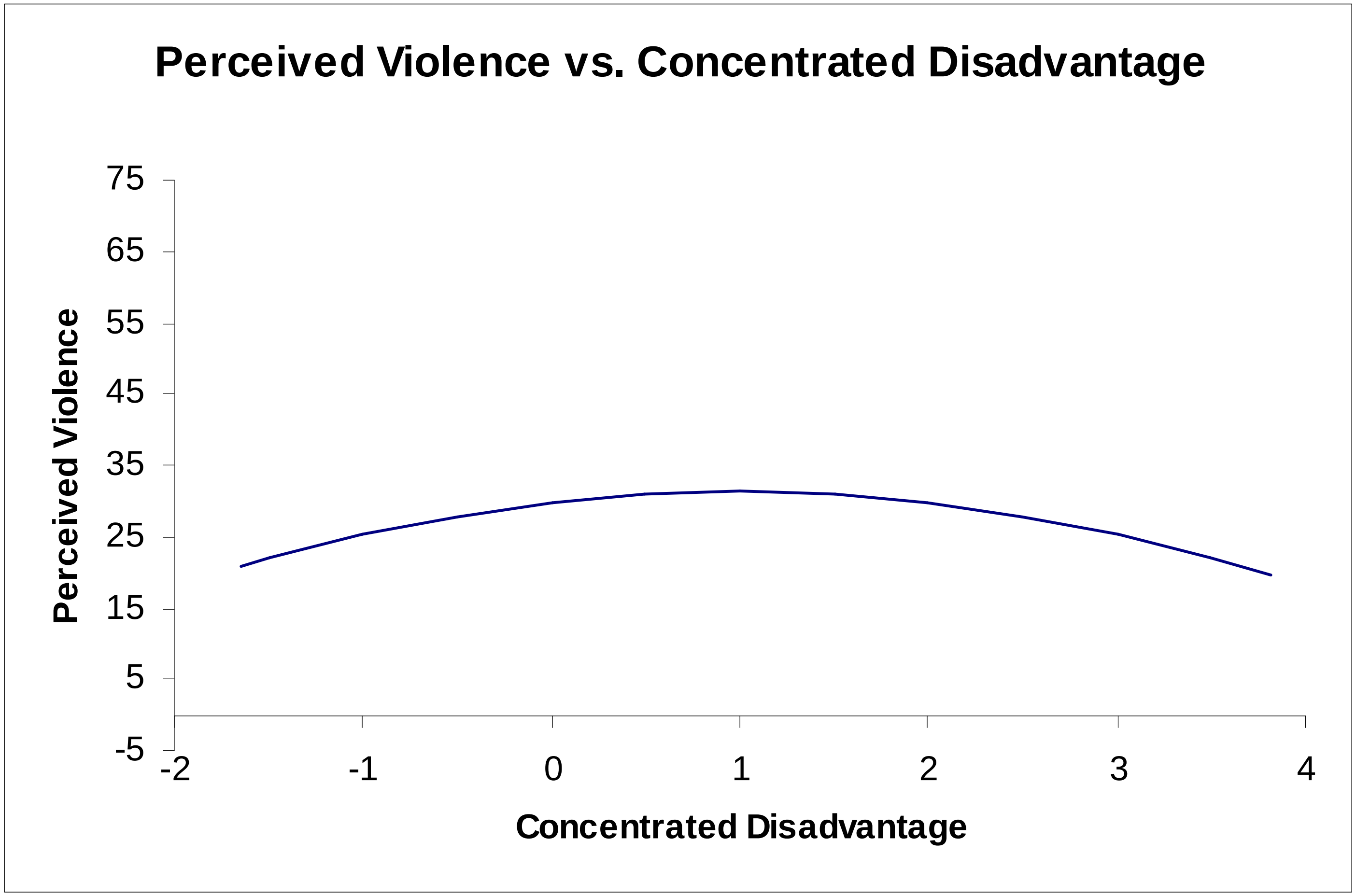
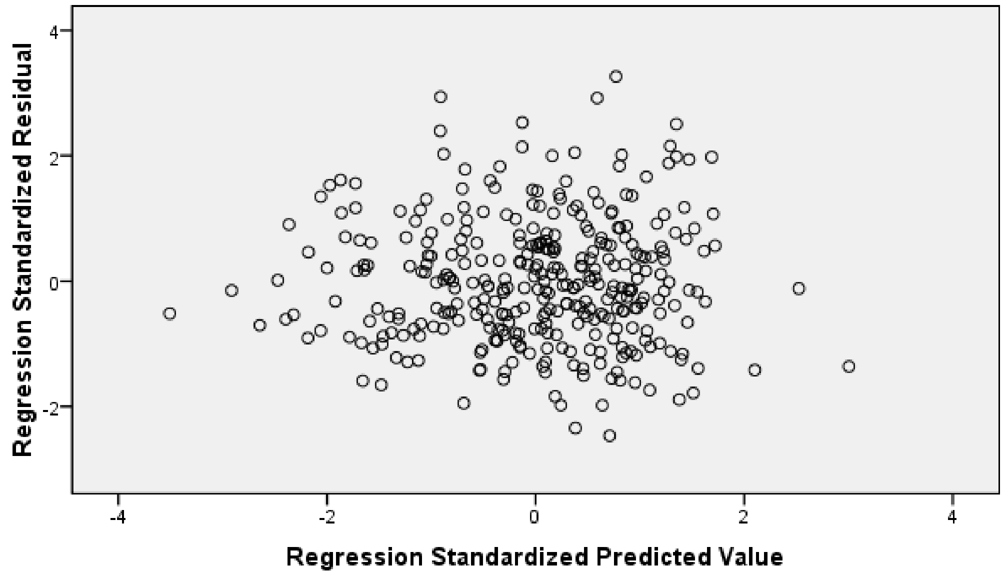


Table 1. **Parameter estimates, (standard errors), approximate p values and goodness-of-fit tests for a nested taxonomy of regression models that describe the relationship between perceived violence and concentrated disadvantage controlling for collective efficacy in Chicago neighborhoods (n=342)**

	Models				
	M1	**M2**	**M3**	**M4**	**M5**
Intercept	28.2***	28.2***	29.1***	29.2***	29.0***
	(0.7)	(0.7)	(0.8)	(0.8)	(0.9)
Concentrated Disadvantage	1.6*	-1.8*	-0.9	-0.7	-0.6
	(0.7)	(0.9)	(1.0)	(1.1)	(1.1)
Collective Efficacy		-5.4***	-4.9***	-5.1***	-4.9***
		(0.9)	(0.9)	(0.9)	(0.9)
Disadvan/Efficacy Interaction			1.4~	(Note that comparison statistics are with M2.)	0.9
			(0.7)		(1.0)
Disadvan/Disadvan Interaction				-1.0~	-0.6
				(0.5)	(0.6)
R^2	0.01*	0.11***	0.12***	0.12***	0.12***
df(Residual)	340	339	338	338	337
ΔR^2		0.10***	0.01~	0.01~	0.00
df (ΔR^2)		1	1	1	1

Key: ~p <.10 *p<.05; **p<.01; ***p<.001

$$\hat{PercViol} = 29.1 - 1.0ZDisadv - 4.8ZCollEff + 1.4ZDisadv * ZCollEff$$



$$\hat{PercViol} = 29.2 - 0.7ZDisadv - 5.1ZCollEff - 1.0ZDisadv * ZDisadv$$